Symbolic Knowledge Extraction and Injection: Theory and Methods

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ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA
DEPARTMENT
OF COMPUTER SCIENCE
AND ENGINEERING

Link to these slides

https://MatteoMagnini.github.io/talk-2025-icr-claim-nesy/

printable version

About me

Research topics

- Symbolic and Neuro-Symbolic AI (NeSy)
  - Symbolic Knowledge Injection (SKI)
  - Symbolic Knowledge Extraction (SKE)
- Explainable AI (XAI)
- Fairness in AI
  - Regularization for Group Fairness
- Large Language Models (LLMs)
  - RAG pipelines
  - Medical applications

Interests

- Scuba Diver
  - NADD ADV
  - ~40 dives
- Chess
  - ELO ~1750 (estimated)
- History
- Hiking
- Cooking
  - (or should I say eating)

Recent updates

ALMA MATER STUDIORUM – UNIVERSITÀ DI BOLOGNA
DISI Dipartimento di Informatica: Scienza e Ingegneria

Dottorato di Ricerca in
Computer Science and Engineering

Ciclo XXXVIII

Settore Scientifico Disciplinare: ING-INF/05
Settore Concorsuale: 09/H1

Symbolic Knowledge Injection & Extraction for Autonomous Learning

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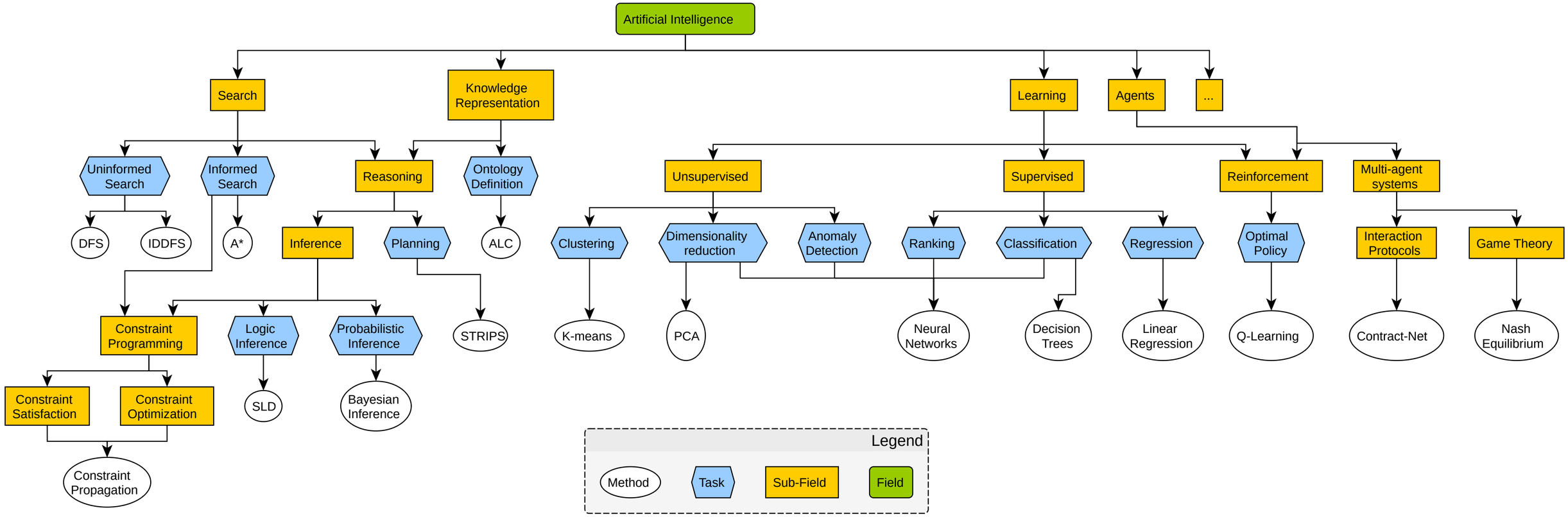
Esame finale anno 2026

Background

Quick overview on symbolic vs. sub-symbolic AI

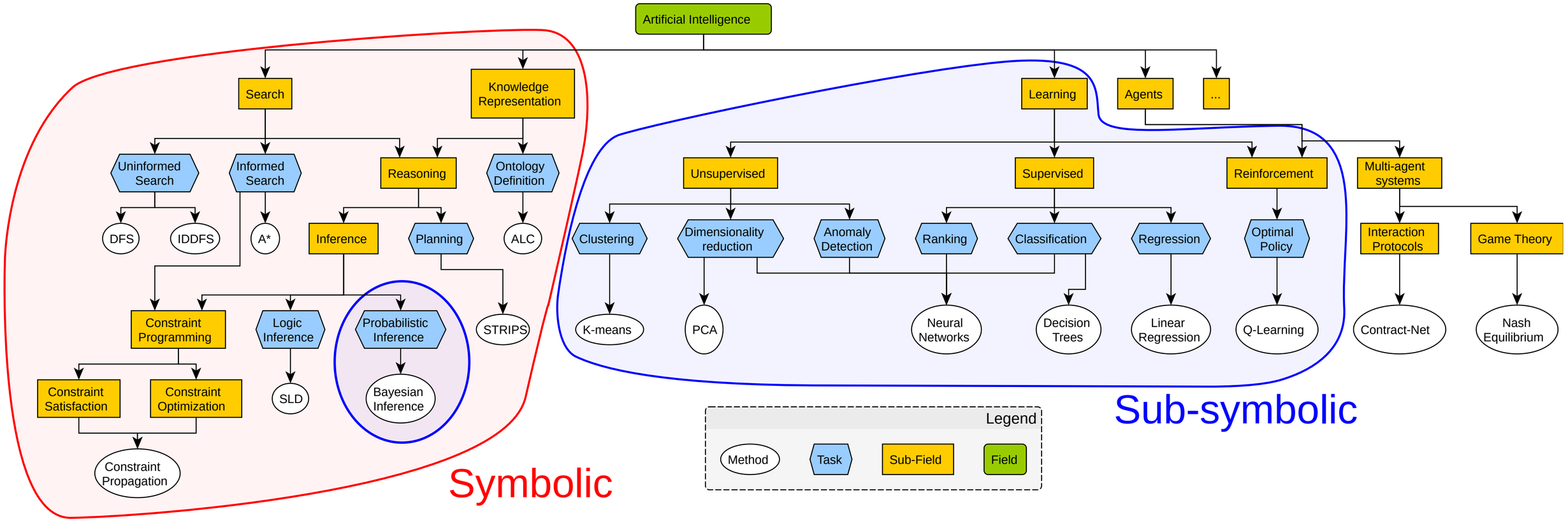
Overview on AI



- wide field of research, with many *sub-fields*
- each sub-field has its own relevant *tasks* (problems) ...
- ... and each task comes with many useful *methods* (algorithms)

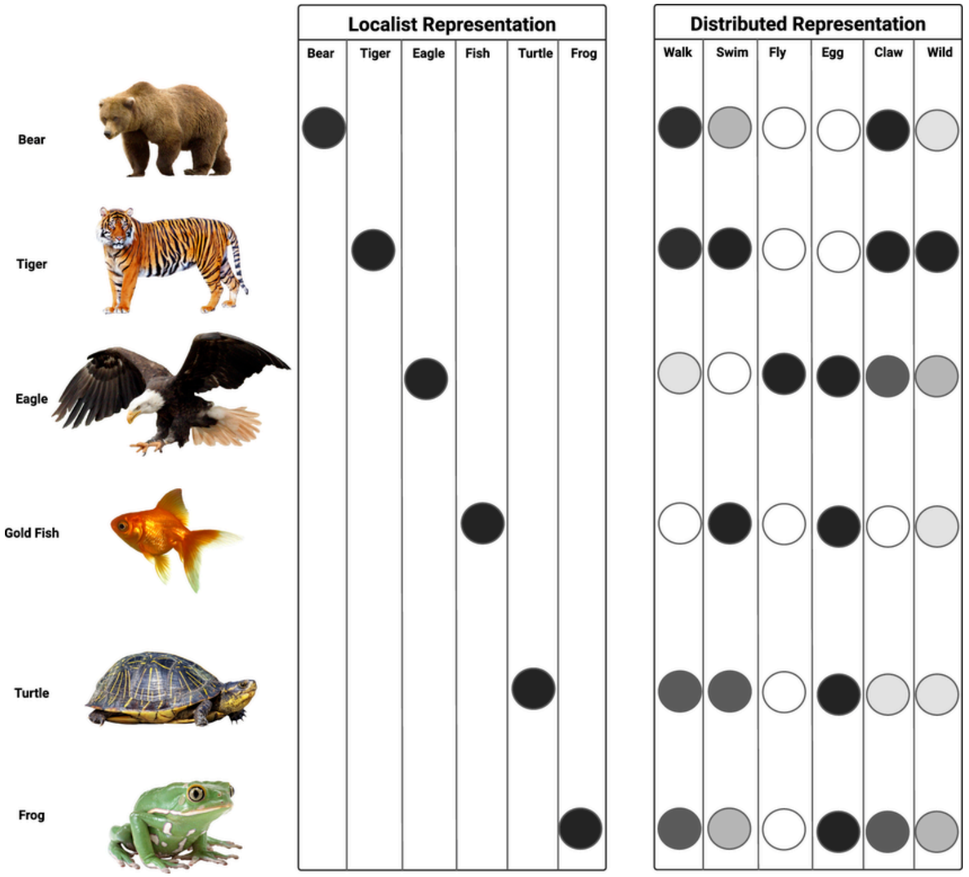
Symbolic vs. Sub-symbolic AI

Two broad categories of AI approaches:



Why the wording “Symbolic” vs. “Sub-symbolic”? (pt. 1)

Local vs. Distributed Representations



- **Local** ≈ “symbolic”: each symbol has a clear, distinct meaning
  - e.g. `"bear"` is a symbol denoting a crisp category (either the animal is a bear or not)
- **Distributed** ≈ “non-symbolic”: symbols do not have a clear meaning per se, but the whole representation does
  - e.g. `"swim"` is fuzzy capability: one animal may be (un)able to swim to some extent

> Let’s say we need to represent N classes, how many columns would the tables have?

Why the wording “Symbolic” vs. “Sub-symbolic”? (pt. 2)

What is a “symbol” after all? Aren’t numbers symbols too?

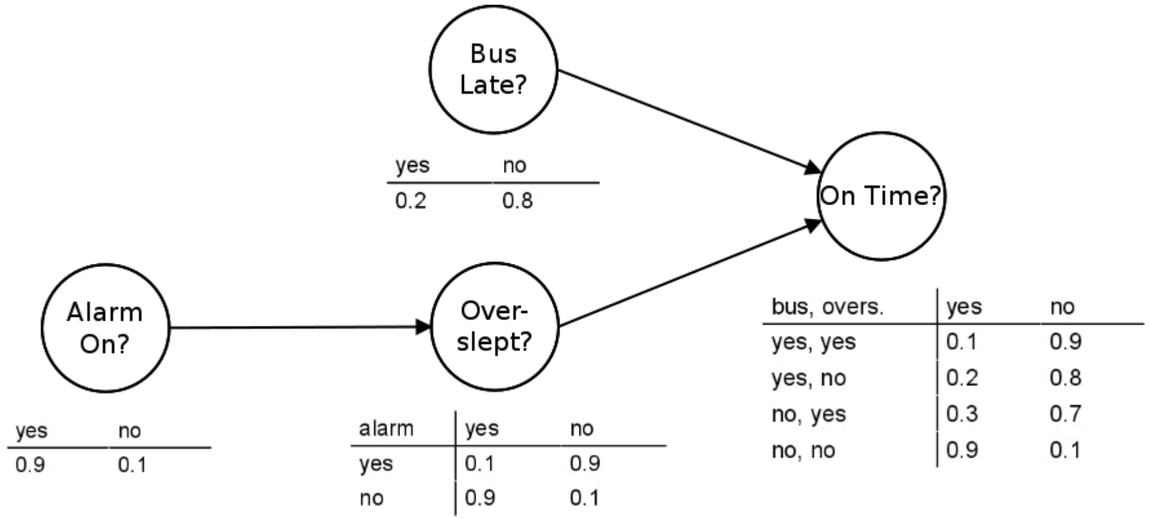
According to Tim van Gelder in 1990:

Symbolic representations of knowledge

- involve a *set of symbols*
- which can be *combined* (e.g., concatenated) in (possibly) infinitely many ways,
- following precise *syntactical rules*,
- where both elementary symbols and any admissible combination of them can be *assigned with meaning*

Why "*Sub*-symbolic" instead of "Non-symbolic" or just "Numerical"?

- There exist approaches where symbols are combined with numbers, e.g.:
  - **Probabilistic logic programming**: where logic statements are combined with probabilities
  - **Fuzzy logic**: where logic statements are combined with degrees of truth
  - **Bayesian networks**: a.k.a. graphical models, where nodes are symbols and edges are conditional dependencies with probabilities, e.g.



- These approaches are *not purely symbolic*, but they are *not purely numeric* either, so we call the overall category **"sub-symbolic"**

Examples of Symbolic AI (pt. 1)

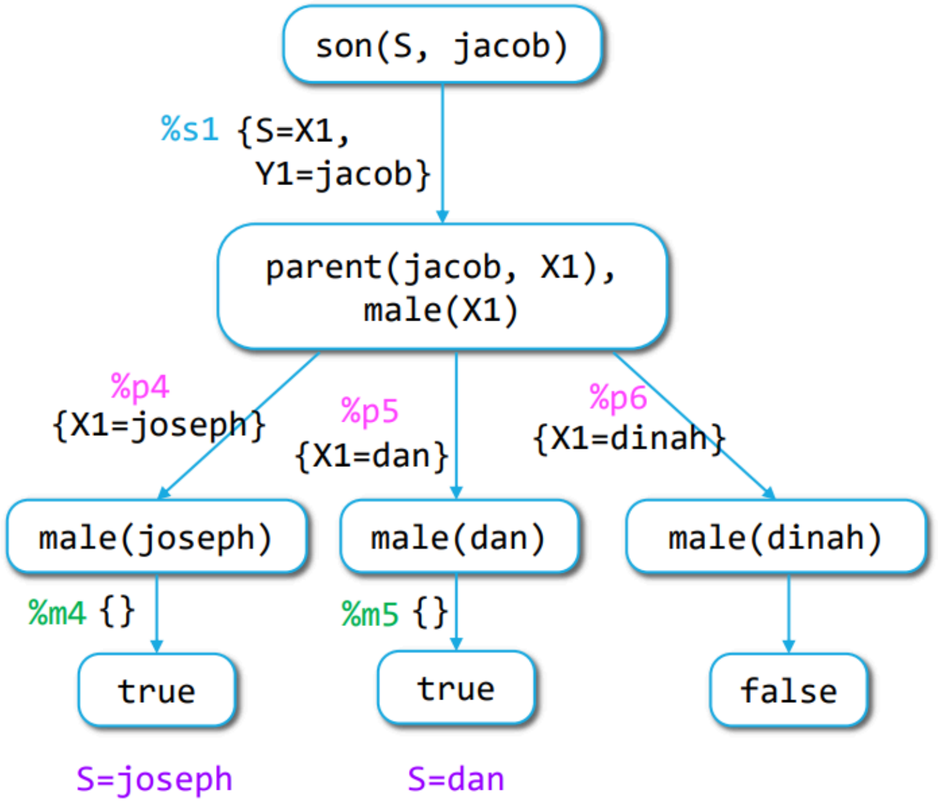
- **Logic programming**: SLD resolution (e.g., Prolog)
- **Knowledge representation**: Semantic Web (e.g., OWL), Description Logics (e.g., ALC)
- **Automated reasoning**: Theorem proving, Model checking
- **Planning**: STRIPS, PDDL

Examples of Symbolic AI (pt. 2)

Logic programming with SLD resolution

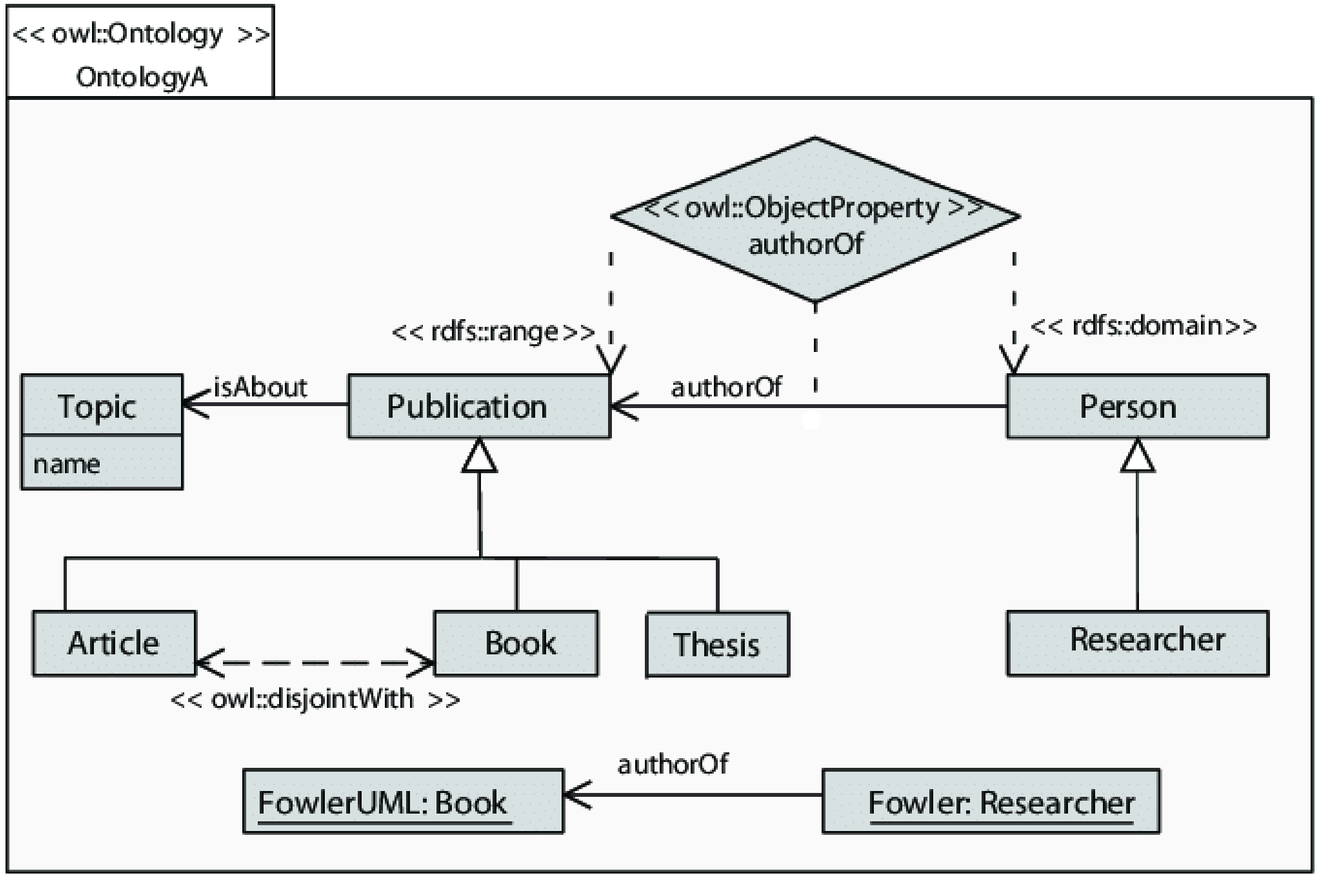
```
parent(abraham, isaac). %p1
parent(isaac, jacob).   %p2
parent(sarah, isaac).   %p3
parent(jacob, joseph).  %p4
parent(jacob, dan).     %p5
parent(jacob, dinah).   %p6

male(abraham).          %m1
male(isaac).            %m2
male(jacob).            %m3
male(joseph).           %m4
male(dan).              %m5

son(X,Y) :- parent(Y,X), %s1
            male(X).
```

```
?- son(S,jacob).
```



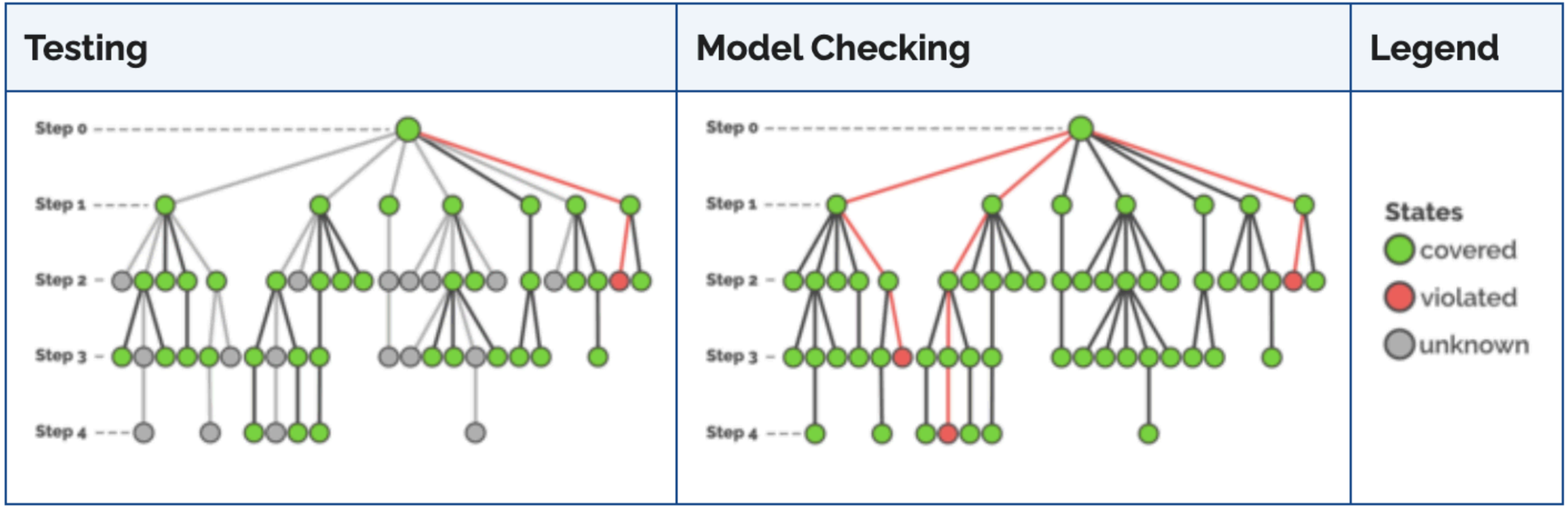
Examples of Symbolic AI (pt. 3)

Ontology definition in OWL



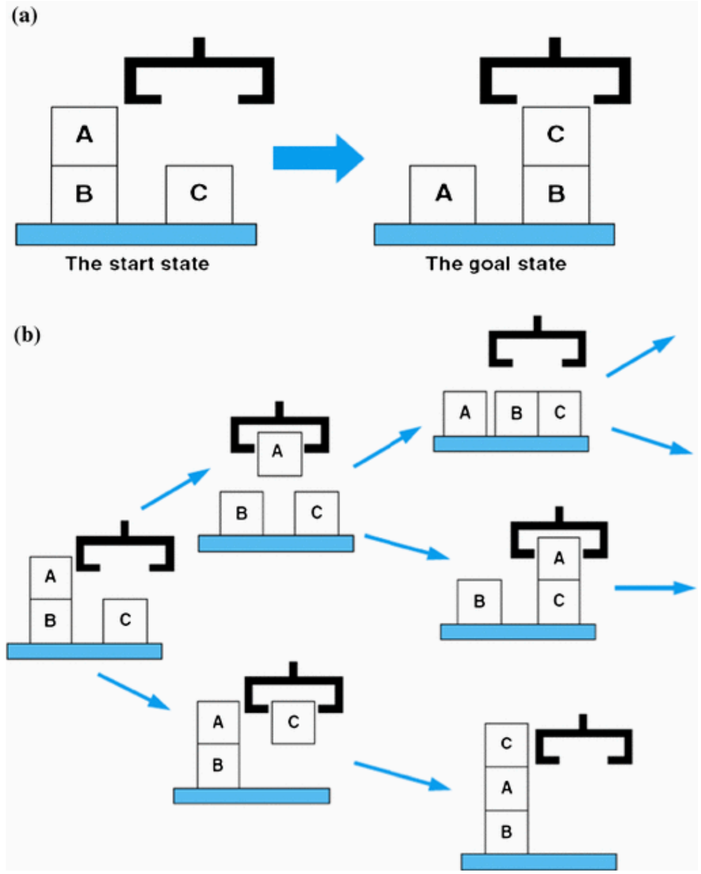
Examples of Symbolic AI (pt. 4)

Model-checking (as opposed to testing)



Examples of Symbolic AI (pt. 5)

Planning in STRIPS



Available actions

- `grab(X)`: grabs block `X` from the table
- `put(X)`: puts block `X` on the table
- `stack(X, Y)`: stacks block `X` on top of block `Y`
- `unstack(X, Y)`: un-stacks block `X` from block `Y`

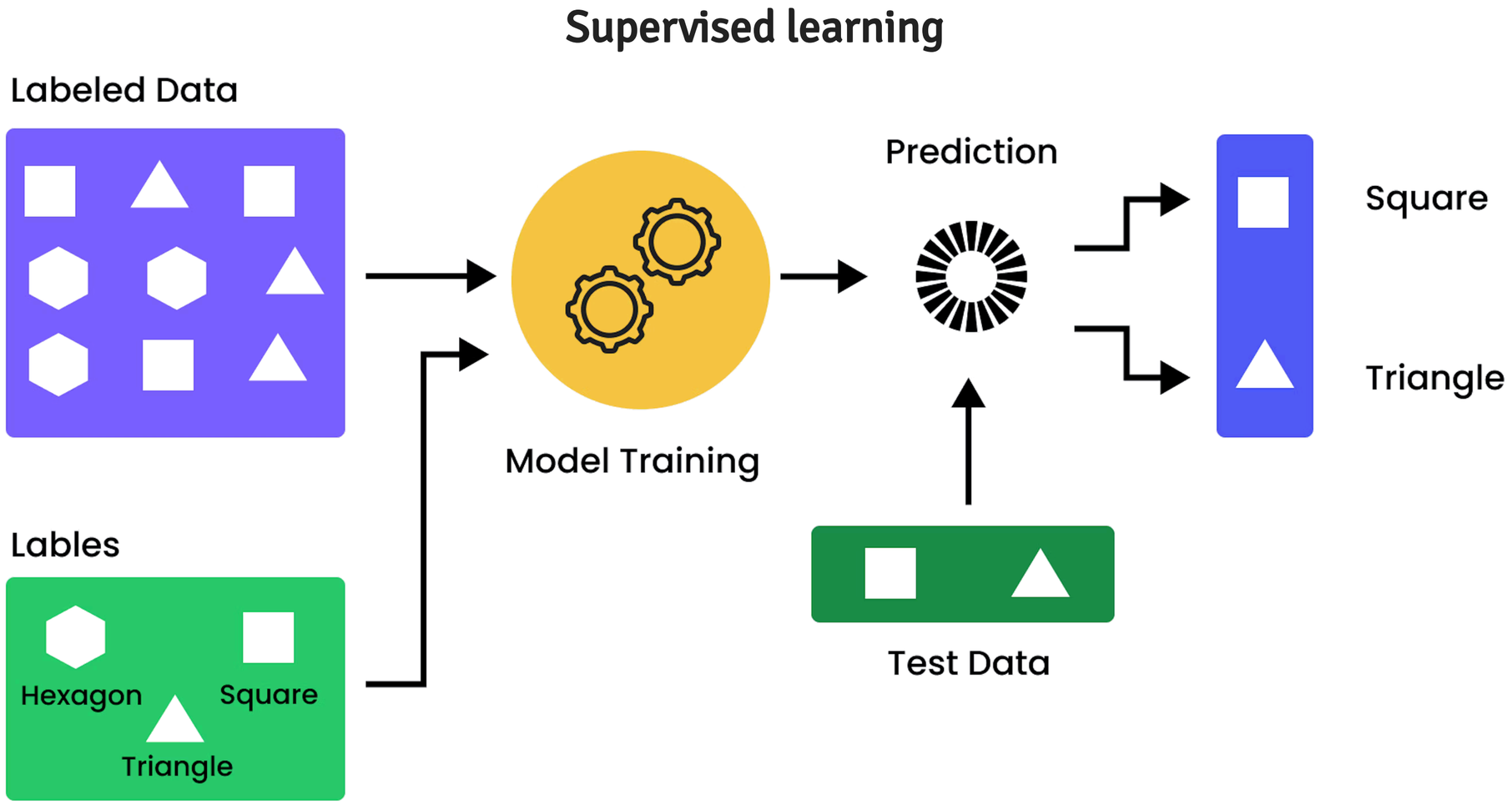
What do these *symbolic* approaches have in common?

- **Structured representations**: knowledge (I/O data) is represented in a structured, formal way (e.g., logic formulas, ontologies)
- **Algorithmic manipulation of representations**: each approach relies on algorithms that manipulate these structured representations following exact rules
- **Crisp semantics**: the meaning of the representations is well-defined, and the algorithms produce exact results
  - representations are either *well-formed or not*, algorithms rely on rules which are either *applicable or not*
- **Model-driven**: algorithms may commonly work in zero- or few-shot settings, humans must commonly model and encode knowledge in the target structure
- **Clear computational complexity**: the decidability, complexity, and tractability of the algorithms are well understood

Examples of Sub-symbolic AI (pt. 1)

- **Machine learning**: supervised, unsupervised, and reinforcement learning
  - *Supervised* learning: fitting a discrete (classification) or a continuous function (regression) from examples
  - *Unsupervised* learning: clustering, dimensionality reduction
  - *Reinforcement* learning: learning a policy to maximize a reward signal, via simulation
- **Probabilistic reasoning**: Bayesian networks, Markov models, probabilistic logic programming

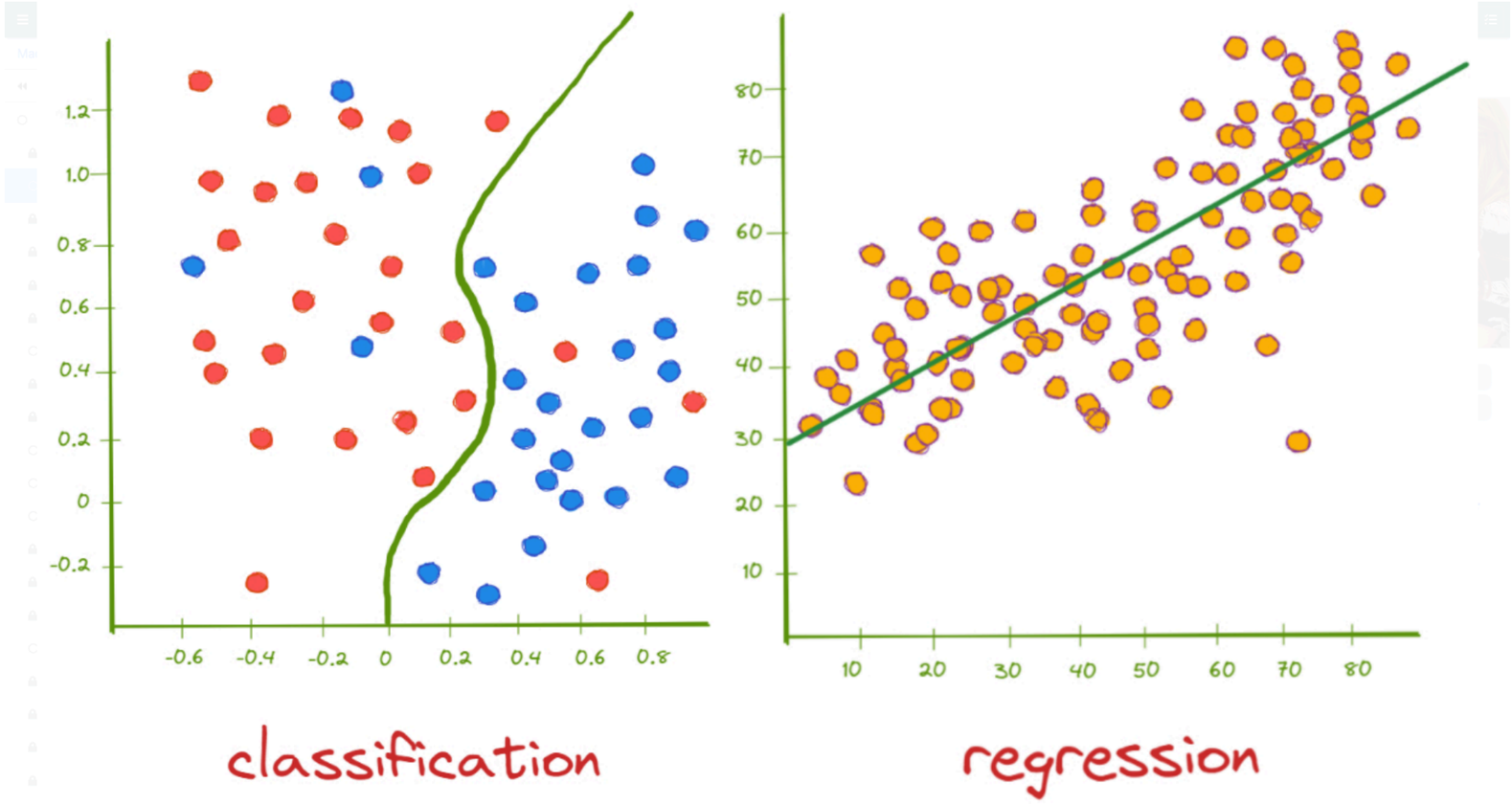
Examples of Sub-symbolic AI (pt. 2)



Examples of Sub-symbolic AI (pt. 3)

Supervised learning – Classification vs. Regression (1/2)

Data separation vs. curve fitting:



Examples of Sub-symbolic AI (pt. 4)

Supervised learning – Classification vs. Regression (2/2)

Focus on the target feature:

Classification Data

X_1	X_2	X_3	X_p	Y
				cat
				dog
				cat
				cat

Categorical "Labels"

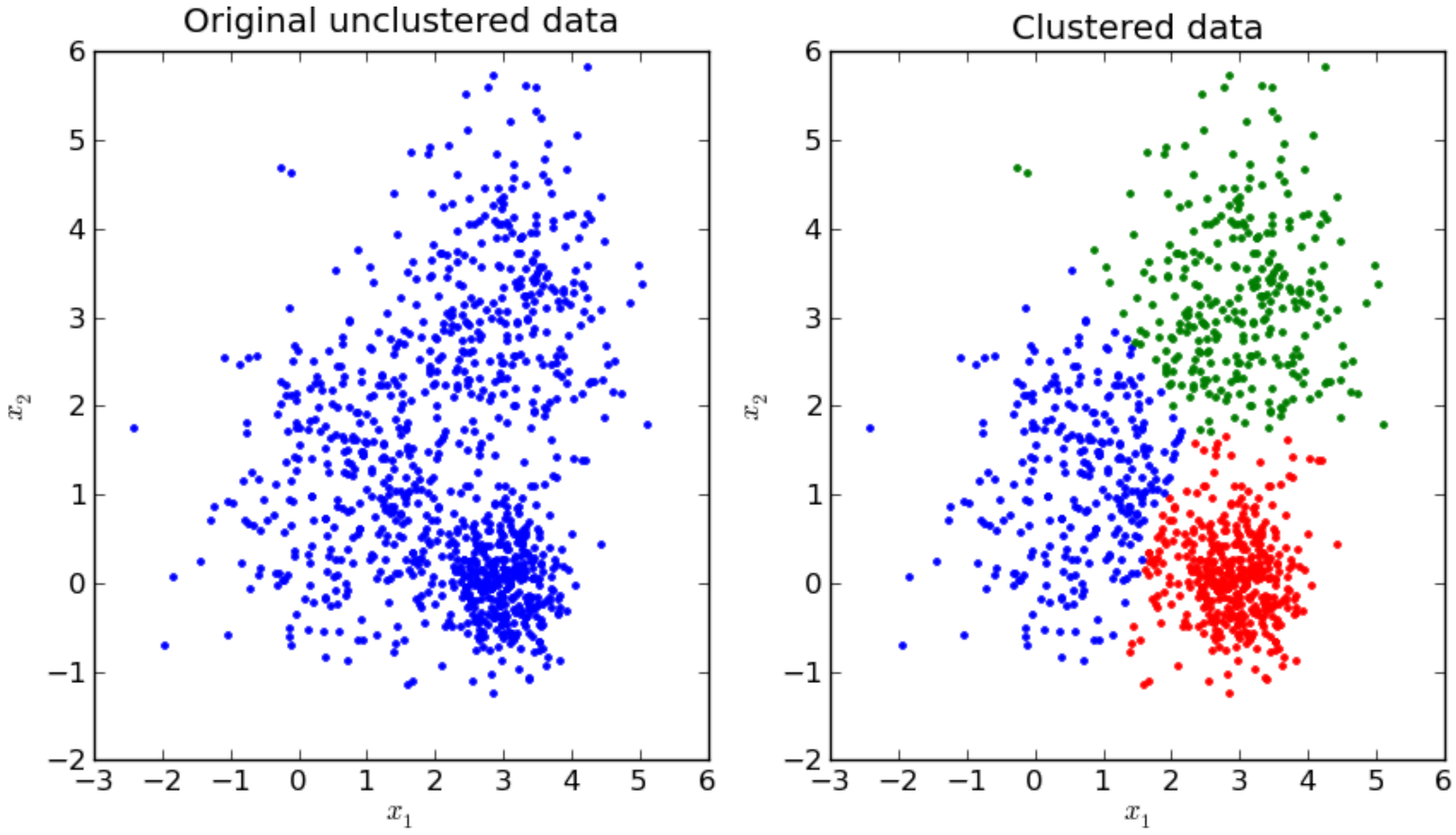
Regression Data

X_1	X_2	X_3	X_p	Y
				5.2
				1.3
				23.0
				7.4

Numeric Target

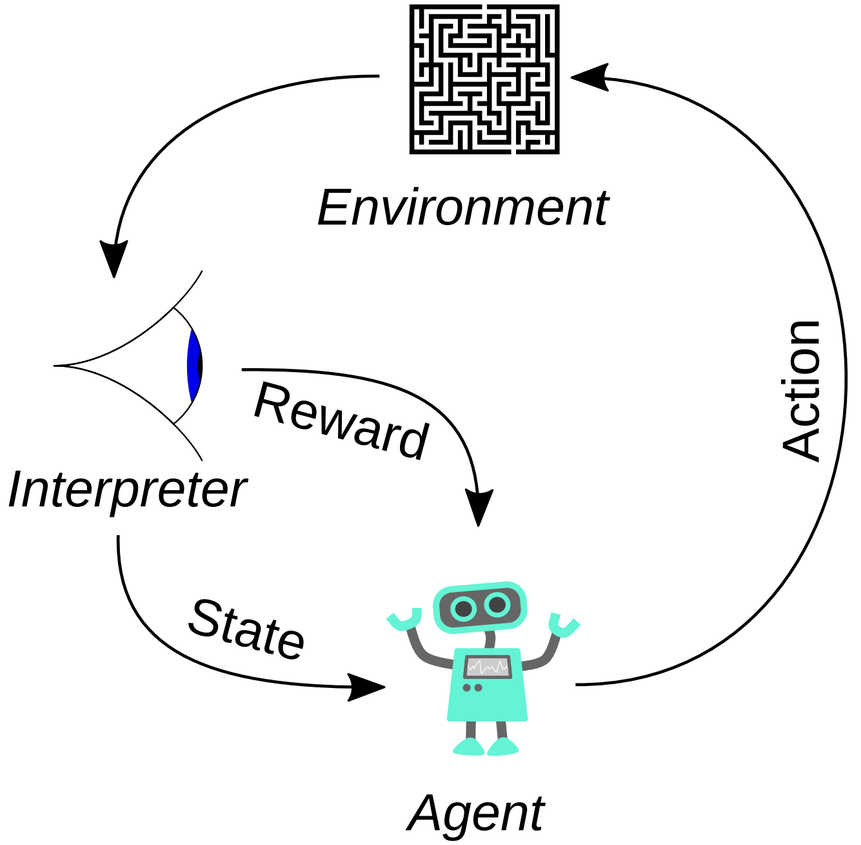
Examples of Sub-symbolic AI (pt. 5)

Unsupervised learning – Clustering



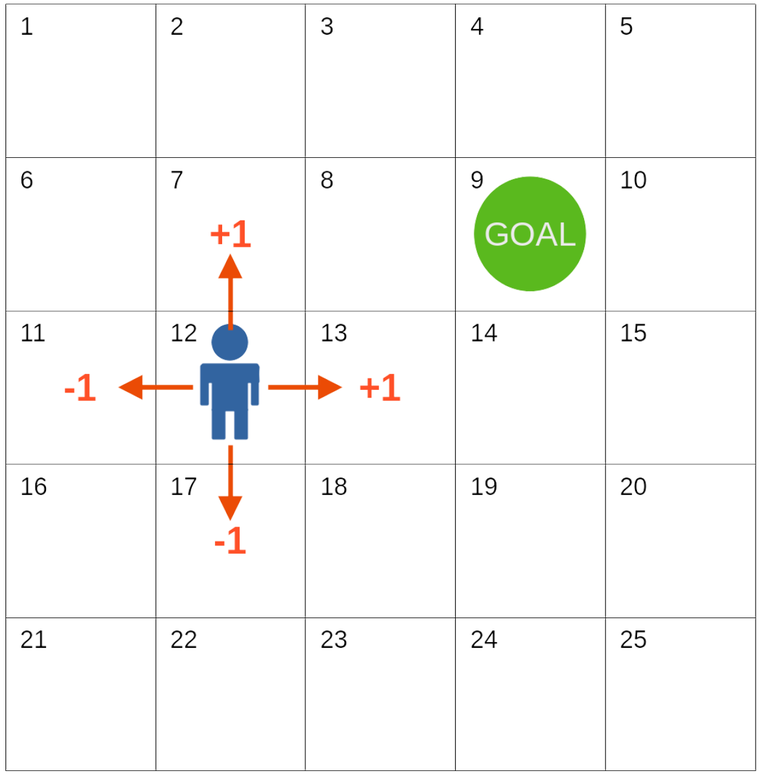
Examples of Sub-symbolic AI (pt. 6)

Unsupervised learning – Reinforcement learning (metaphor)



Examples of Sub-symbolic AI (pt. 7)

Reinforcement learning – Reinforcement learning (policy)



	↑	↓	←	→
1	-	+1	-	+1
2	-	+1	-1	+1
3	-	+1	-1	+1
4	-	+1	-1	-1
5	-	+1	+1	-
...				
23	+1	-	-1	+1
24	+1	-	-1	-1
25	+1	-	+1	-

What do these *sub-symbolic* approaches have in common?

- **Numeric representations**: knowledge (I/O data) is represented in a less structured way, often as vectors/matrices/tensors of numbers
- **Differentiable manipulation of representations**: algorithms rely on mathematical operations involving these numeric representations, most-commonly undergoing some optimization process
  - e.g., sum, product, max, min, etc.
- **Fuzzy/continuous semantics**: representations are from continuous spaces, where similarities and distances are defined in a continuous way, and algorithms may yield fuzzy results
- **Data-driven** + **Usage vs. training**: algorithms are often trained on data, to be later re-used on other data
  - usage is commonly impractical or impossible without training
- **Unclear computational complexity**: strong reliance on greedy or time-limited optimization methods, lack of theoretical guarantees on the quality of the results

Long-standing dualism

Intuition vs. Reasoning

1. Esprit de *finesse* vs. Esprit de *géométrie* (Philosophy) — Blaise Pascal, 1669
2. *Cognitive* vs. *Behavioural* Psychology — B.F. Skinner, 1950s
3. *System 1* (fast, intuitive) vs. *System 2* (slow, rational) — Daniel Kahneman, 2011

Sub-symbolic AI

- Provides mechanisms emulating human-like *intuition*
- *Quick*, possibly *error-prone*, but often *effective*
- Requires *learning* from data
- Often *opaque*, hard to interpret or explain

Symbolic AI

- Provides mechanisms emulating human-like *reasoning*
- *Slow*, but *precise* and *verifiable*
- Requires symbolic *modeling* and *encoding* knowledge
- Often *transparent*, easier to interpret and explain

Need for integration

- the NeSy community has long recognized the *complementarity* among symbolic and sub-symbolic approaches...
- ... with a focus on **neural-networks** (*NN*) based sub-symbolic methods, as they are very *flexible*

Patterns of *integration* or *combination* (cf. Bhuyan et al., 2024)

1. `Symbolic Neuro-Symbolic`: symbols $\rightarrow$ vectors $\rightarrow$ NNs $\rightarrow$ vectors $\rightarrow$ symbols
2. `Symbolic[Neuro]`: symbolic module $\xrightarrow{invokes}$ NN $\rightarrow$ output
3. `Neuro | Symbolic`: NN $\xrightarrow{cooperates}$ symbolic module $\xrightarrow{cooperates}$ NN $\rightarrow$ ...
4. `Neuro-Symbolic → Neuro`: symbolic knowledge $\xrightarrow{influences}$ NN
5. $\texttt{Neuro}_{\texttt{Symbolic}}$: symbolic knowledge $\xrightarrow{constrains}$ NN
6. `Neuro[Symbolic]`: symbolic module $\xrightarrow{embedded\ in}$ NN

Focus on two main approaches

(cf. Ciatto et al., 2024)

- Symbolic Knowledge **Extraction** (*SKE*): extracting symbolic knowledge from sub-symbolic models
  - for the sake of *explainability* and *interpretability* in machine learning
- Symbolic Knowledge **Injection** (*SKI*): injecting symbolic knowledge into sub-symbolic models
  - for the sake of *trustworthiness* and *robustness* in machine learning

Both require some basic understanding of how *supervised* **machine learning** works

Symbolic Knowledge Extraction (SKE)

How to extract symbolic knowledge from sub-symbolic predictors

Definition and Motivation (pt. 1)

any *algorithmic procedure* accepting trained sub-symbolic predictors as input and producing *symbolic* knowledge as output, so that the extracted knowledge reflects the behaviour of the predictor with high *fidelity*.

Definition and Motivation (pt. 2)

- **Explainable AI (XAI)**: SKE methods are often used to provide explanations for the decisions made by sub-symbolic predictors, making them more interpretable and understandable to humans (a.k.a. *post-hoc explainability*)
  - *local explanations*: explanations for individual predictions
  - *global explanations*: explanations for the overall behaviour of the predictor
- **Knowledge discovery**: SKE methods can help discover patterns and relationships in the data that may not be immediately apparent, thus providing insights into the underlying processes
- **Model compression**: SKE methods can simplify complex sub-symbolic models by extracting symbolic rules that approximate their behaviour, thus reducing the model's size and complexity

Explainability vs Interpretability

They are *not* synonyms in spite of the fact that they are often used interchangeably!

Explanation

- elicits *relevant aspects* of objects (to ease their interpretation)
- it is an operation that transform poorly interpretable objects into *more interpretable* ones
- search of a *surrogate* interpretable model

Interpretation

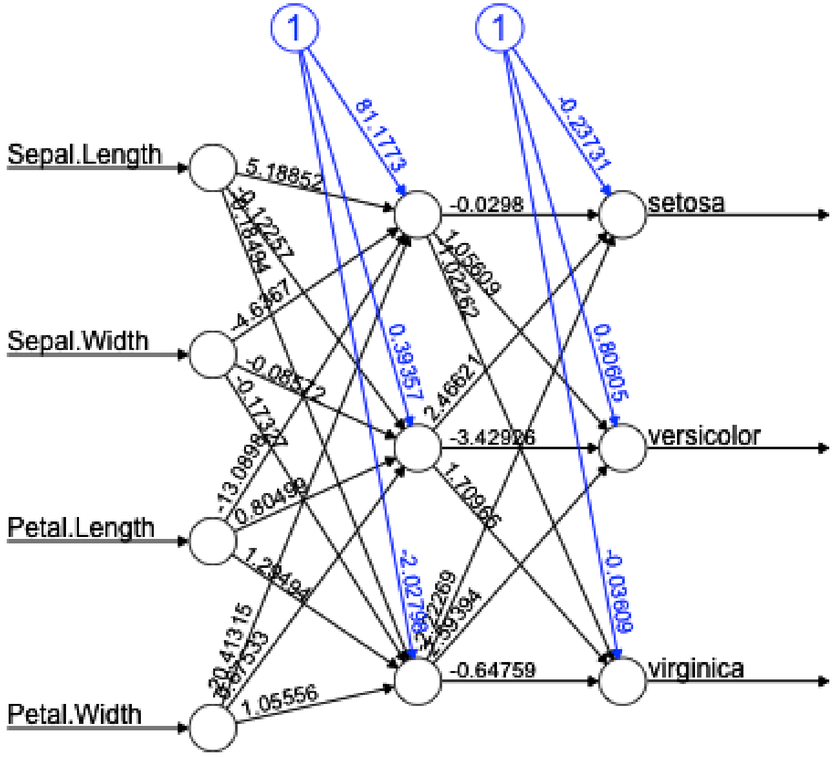
- binds objects with *meaning* (what the human mind does)
- it is *subjective*
- it does not need to be measurable, only *comparisons*

Concepts

Main entities and how to extract symbolic knowledge from sub-symbolic predictors

Entities

Sub-symbolic predictor



Error: 0.346668 Steps: 26926

Symbolic knowledge

Logic Rule
Class = setosa ← PetalWidth ≤ 1.0
Class = versicolor ← PetalLength > 4.9 ∧ SepalWidth ∈ [2.9, 3.2]
Class = versicolor ← PetalWidth > 1.6
Class = virginica ← SepalWidth ≤ 2.9
Class = virginica ← SepalLength ∈ [5.4, 6.3]
Class = virginica ← PetalWidth ∈ [1.0, 1.6]

How SKE works

Decompositional SKE

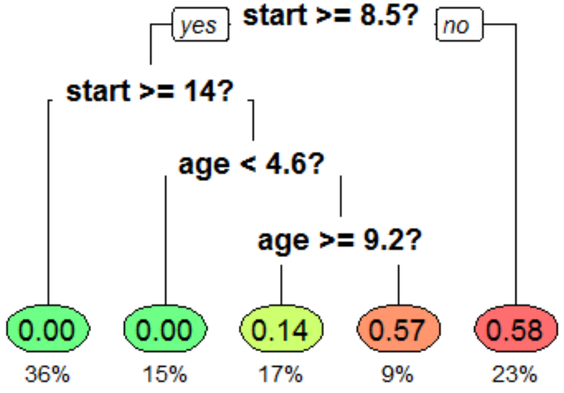
if the method *needs* to inspect (even partially) the internal parameters of the underlying black-box predictor, e.g., neuron biases or connection weights for NNs, or support vectors for SVMs

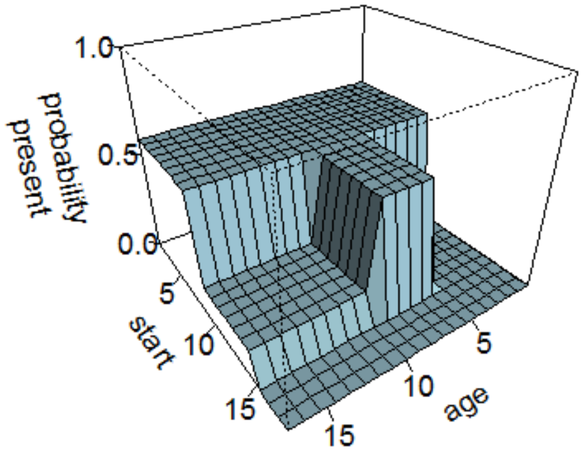
Pedagogical SKE

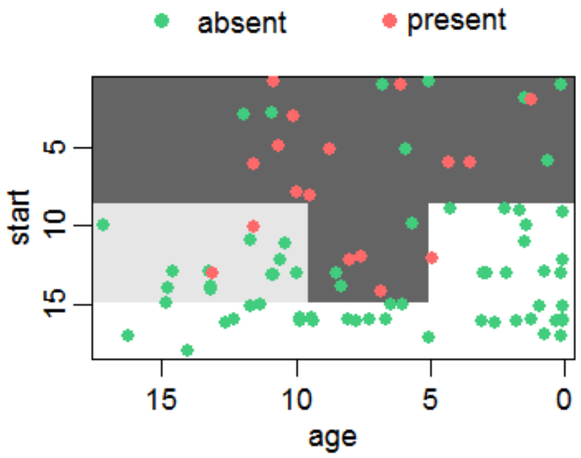
if the algorithm *does not need* to take into account any internal parameter, but it can extract symbolic knowledge by only relying on the predictor's outputs.

CART (pt. 1)

Classification and regression trees (cf. Breiman et al., 1984)







An example decision tree estimating the probability of kyphosis after spinal surgery, given the *age* of the patient and the vertebra at which surgery was *start* ed (rf. wiki:dt-learning). Notice that all decision trees subtend a partition of the input space, and that those trees themselves provide intelligible representations of *how* predictions are attained.

CART (pt. 2)

1. generate a *synthetic* dataset by using the predictions of the sub-symbolic predictor
2. *train* a decision tree on the synthetic dataset
3. compute the *fidelity* and repeat step 2 until satisfied
4. [optional] rewrite the tree as a set of symbolic *rules*

Adult classification task (pt. 1)

The Adult dataset (cf. Becker Barry and Kohavi Ronny, 1996) contains the records (48,842) of individuals based on census data (this dataset is also known as Census Income). The dataset has many features (14) related to the individuals' demographics, such as age, education, and occupation. The target feature is whether the individual earns more than `$50,000` per year.

Examples of Adult records

age	workclass	education	...	hours-per-week	native-country	income
39	State-gov	Bachelors	...	40	United-States	<=50K
50	Self-emp-not-inc	Bachelors	...	13	United-States	<=50K
38	Private	HS-grad	...	40	United-States	<=50K
53	Private	11th	...	40	United-States	<=50K
28	Private	Bachelors	...	40	Cuba	<=50K
37	Private	Masters	...	40	United-States	<=50K
49	Private	9th	...	16	Jamaica	<=50K
52	Self-emp-not-inc	HS-grad	...	45	United-States	>50K
31	Private	Masters	...	50	United-States	>50K
42	Private	Bachelors	...	40	United-States	>50K

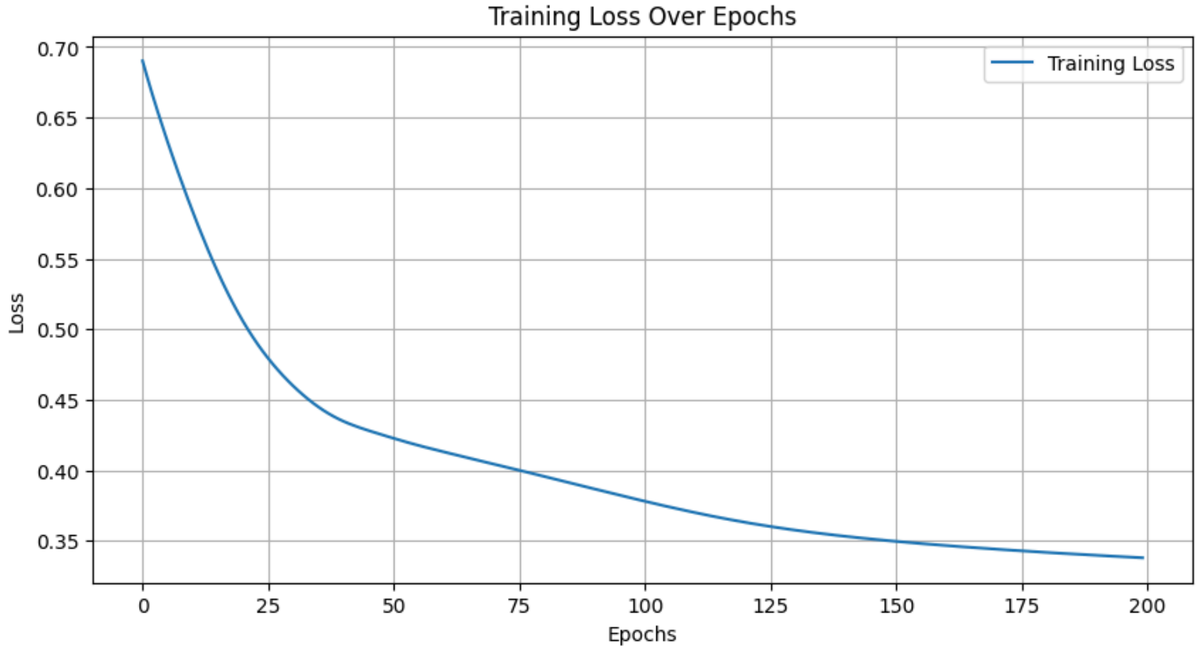
Adult classification task (pt. 2)

We can train a simple feed-forward neural network for a fixed amount of epoches on the Adult dataset to classify whether an individual earns more than `$50,000` per year.

```python
class AdultNet(nn.Module):
    def __init__(self):
        super().__init__()
        self.model = nn.Sequential(
            nn.Linear(FEATURE_NUMBER, HIDDEN_SIZE),
            nn.ReLU(),
            nn.Linear(HIDDEN_SIZE, HIDDEN_SIZE),
            nn.ReLU(),
            nn.Linear(HIDDEN_SIZE, CLASS_NUMBER)
        )

    def forward(self, x):
        return self.model(x)
```

```python
def train_model() -> tuple[nn.Module, list[float]]:
    model = AdultNet()
    model.to(device)
    optimizer = optim.Adam(model.parameters(), lr=0.001)
    criterion = nn.CrossEntropyLoss()
    train_losses = []
    for epoch in range(EPOCHES):
        model.train()
        optimizer.zero_grad()
        output = model(X_train_tensor)
        loss = criterion(output, y_train_tensor)
        loss.backward()
        optimizer.step()
        train_losses.append(loss.item())
        if (epoch + 1) % 10 == 0 or epoch == EPOCHES - 1:
            print(f"Epoch {epoch+1}: loss = {loss.item():.4f}")
    return model, train_losses
```

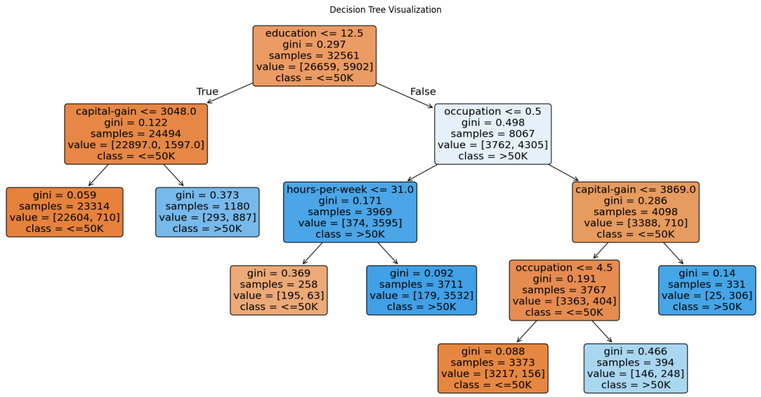
Adult classification task (pt. 3)



SciKitLearn classification report

Class	Precision	Recall	F1-Score	Support
<=50K	0.867812	0.935882	0.900562	24.720
>50K	0.731447	0.550568	0.628247	7.841
Accuracy			0.843094	32.561
Macro Avg	0.799629	0.743225	0.764405	32.561
Weighted Avg	0.834974	0.843094	0.834986	32.561

Extracted rules (pt. 1)




```
Extracted Symbolic Rules:

|--- education <= 12.50
|   |--- capital-gain <= 3048.00
|   |   |--- class: 0
|   |--- capital-gain >  3048.00
|   |   |--- class: 1
|--- education >  12.50
|   |--- occupation <= 0.50
|   |   |--- hours-per-week <= 31.00
|   |   |   |--- class: 0
|   |   |--- hours-per-week >  31.00
|   |   |   |--- class: 1
|   |--- occupation >  0.50
|   |   |--- capital-gain <= 3869.00
|   |   |   |--- occupation <= 4.50
|   |   |   |   |--- class: 0
|   |   |   |--- occupation >  4.50
|   |   |   |   |--- class: 1
|   |   |--- capital-gain >  3869.00
|   |   |   |--- class: 1
```

Decision Rules

1. **class = 0** if `education ≤ 12.5` and `capital-gain ≤ 3048`
2. **class = 1** if `education ≤ 12.5` and `capital-gain > 3048`
3. **class = 0** if `education > 12.5` and `occupation ≤ 0.5` and `hours-per-week ≤ 31`
4. **class = 1** if `education > 12.5` and `occupation ≤ 0.5` and `hours-per-week > 31`
5. **class = 0** if `education > 12.5` and `occupation > 0.5` and `capital-gain ≤ 3869` and `occupation ≤ 4.5`
6. **class = 1** if `education > 12.5` and `occupation > 0.5` and `capital-gain ≤ 3869` and `occupation > 4.5`
7. **class = 1** if `education > 12.5` and `occupation > 0.5` and `capital-gain > 3869`

Extracted rules (pt. 2)

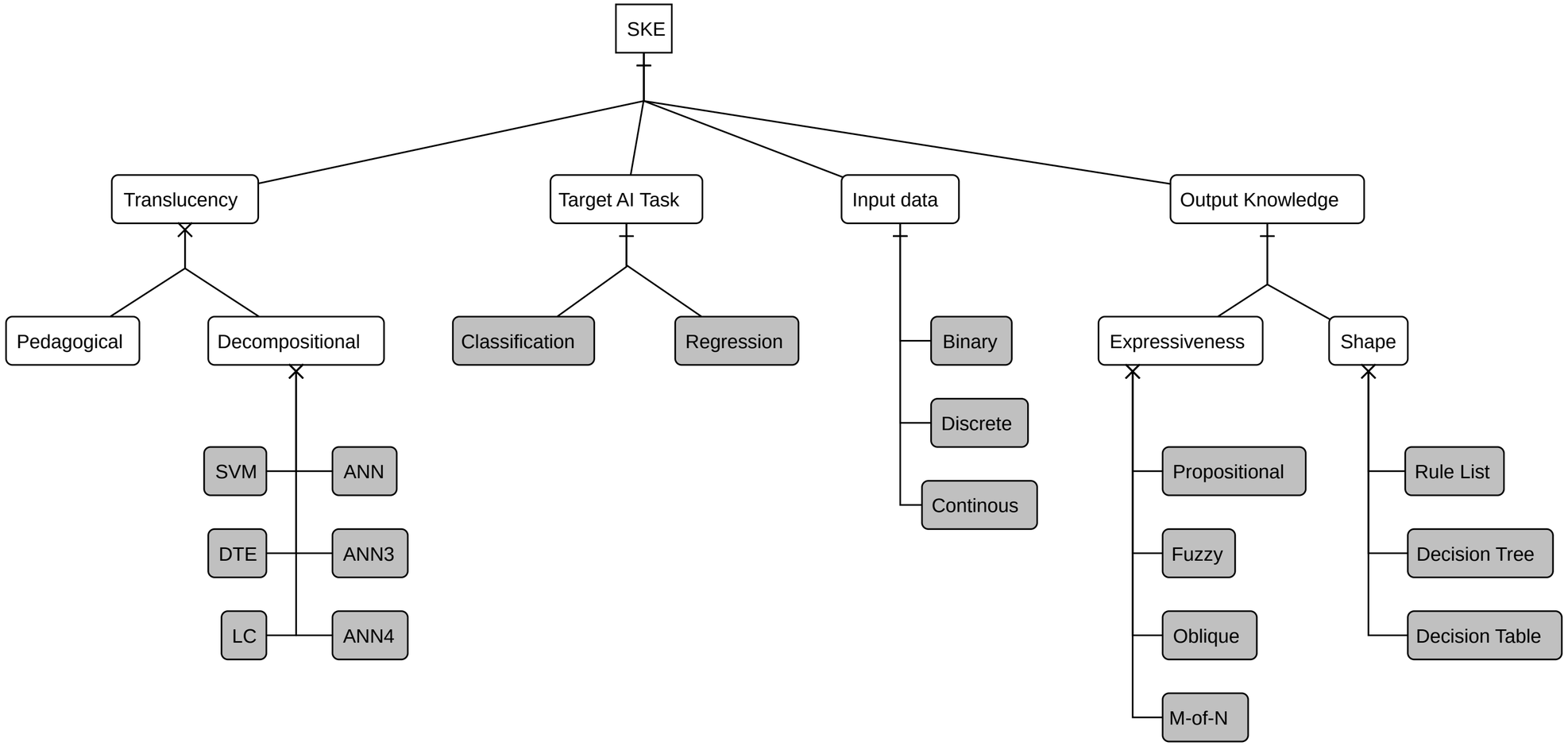
Fidelity of the symbolic predictor

Class	Precision	Recall	F1-Score	Support
0	0.97	0.98	0.97	26659
1	0.89	0.84	0.86	5902
Accuracy			0.95	32561
Macro Avg	0.93	0.91	0.92	32561
Weighted Avg	0.95	0.95	0.95	32561

Jupyter notebook available here

github.com/MatteoMagnini/demo-2025-woa-nesy/blob/master/notebook/extraction.ipynb

Taxonomy of SKE methods (pt. 1)



Taxonomy of SKE methods (pt. 2)

Target AI task

- *classification*
  $f : \mathscr{X} \subseteq \mathbb{R}^n \rightarrow \mathscr{Y} s.t. |\mathscr{Y}| = k$
- *regression*
  $f : \mathscr{X} \subseteq \mathbb{R}^n \rightarrow \mathscr{Y} \subseteq \mathbb{R}^m$

Input data

- *binary*
  $\mathscr{X} \equiv 0, 1^n$
- *discrete*
  $\mathscr{X} \in x_1, \ldots, x_{n^n}$
- *continuous*
  $\mathscr{X} \subseteq \mathbb{R}^n$

Taxonomy of SKE methods (pt. 3)

Shape

- *rule list*, ordered sequences of if-then-else rules
- *decision tree*, hierarchical set of if-then-else rules involving a comparison among a variable and a constant
- *decision table*, 2D tables summarising decisions for each possible assignment of the input variables

Expressiveness

- *propositional*, boolean statements + logic connectives, including arithmetic comparisons among variables and constants
- *fuzzy*, hierarchical set of if-then-else rules involving a comparison among a variable and a constant
- *oblique*, boolean statements + logic connectives + arithmetic comparisons
- *M-of-N*, any of the above + statements of the form “at least k of the following statements are true”

Discussion

Notable remarks

- discretisation of the input space
- discretisation of the output space
- features should have semantic meaning
- rules constitutes global explanations

Limitations

- many methods for tabular data as input, very few for images
- high dimensional datasets could lead to poorly readable rules
- high variable input spaces could do the same

Symbolic Knowledge Injection (SKI)

How to inject symbolic knowledge into sub-symbolic predictors

Definition and Motivation (pt. 1)

Any algorithmic procedure affecting how sub-symbolic predictors draw their inferences in such a way that predictions are either *computed* as a function of, or *made consistent* with, some given symbolic knowledge.

Definition and Motivation (pt. 2)

- **Improve predictive performance**: by injecting symbolic knowledge, we can
  - *guide* the learning process in order to *penalise* inconsistencies with the symbolic knowledge, or
  - *structure* the model's architecture to *mimic* the symbolic knowledge
- **Enhance interpretability**: with SKI we can make predictors that are
  - interpretable by *transparent box design*, as they are built to mimic symbolic knowledge
  - interpretable using *symbols as constraints*, as they are built to respect symbolic knowledge
- **Robustness to data degradation**: symbolic knowledge can help sub-symbolic models maintain performance even in the presence of noisy or scarcity of data
- **Enhance fairness**: by incorporating symbolic knowledge about fairness constraints, we can ensure that sub-symbolic models make decisions that align with ethical considerations
- **And more**: SKI can simplify the predictor's architecture, in particular it can reduce the number of weights in a neural network, thus improving its efficiency and reducing the risk of overfitting

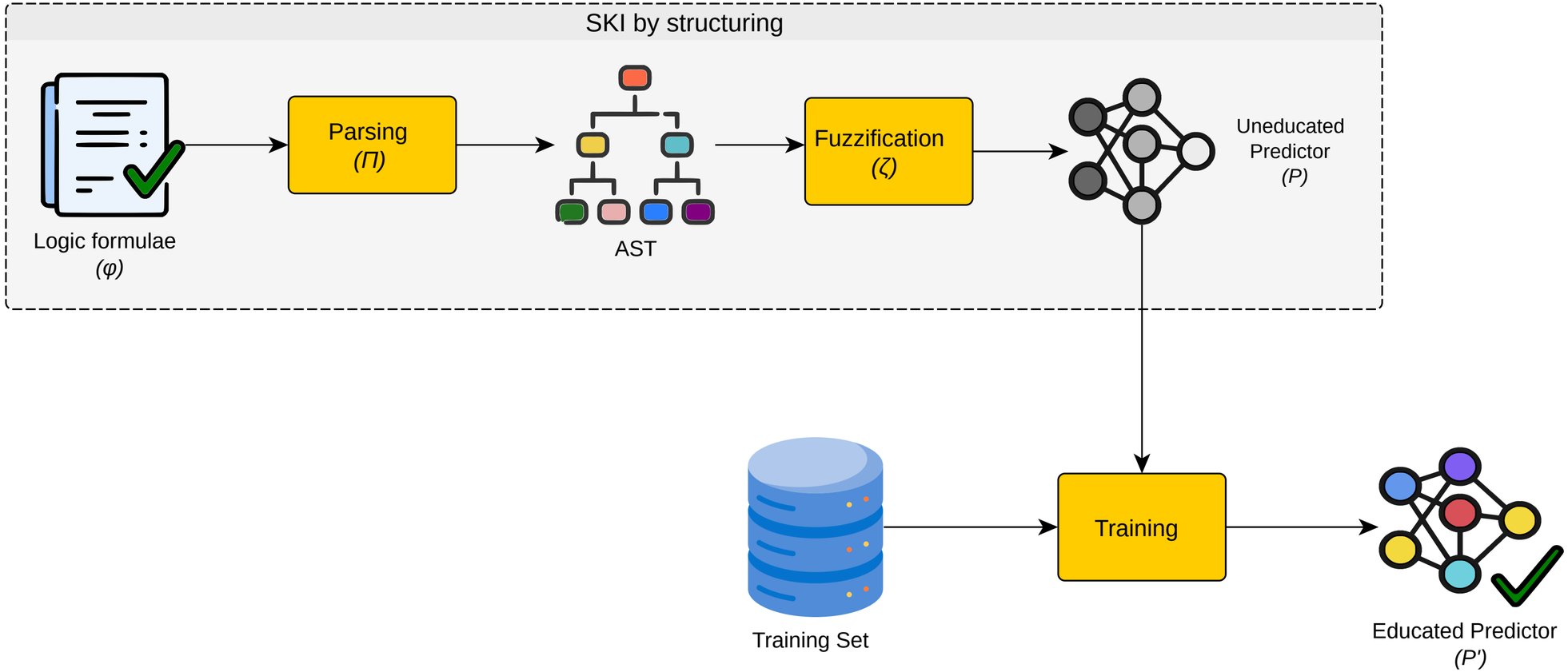
Concepts

Main entities and how to inject symbolic knowledge into sub-symbolic predictors

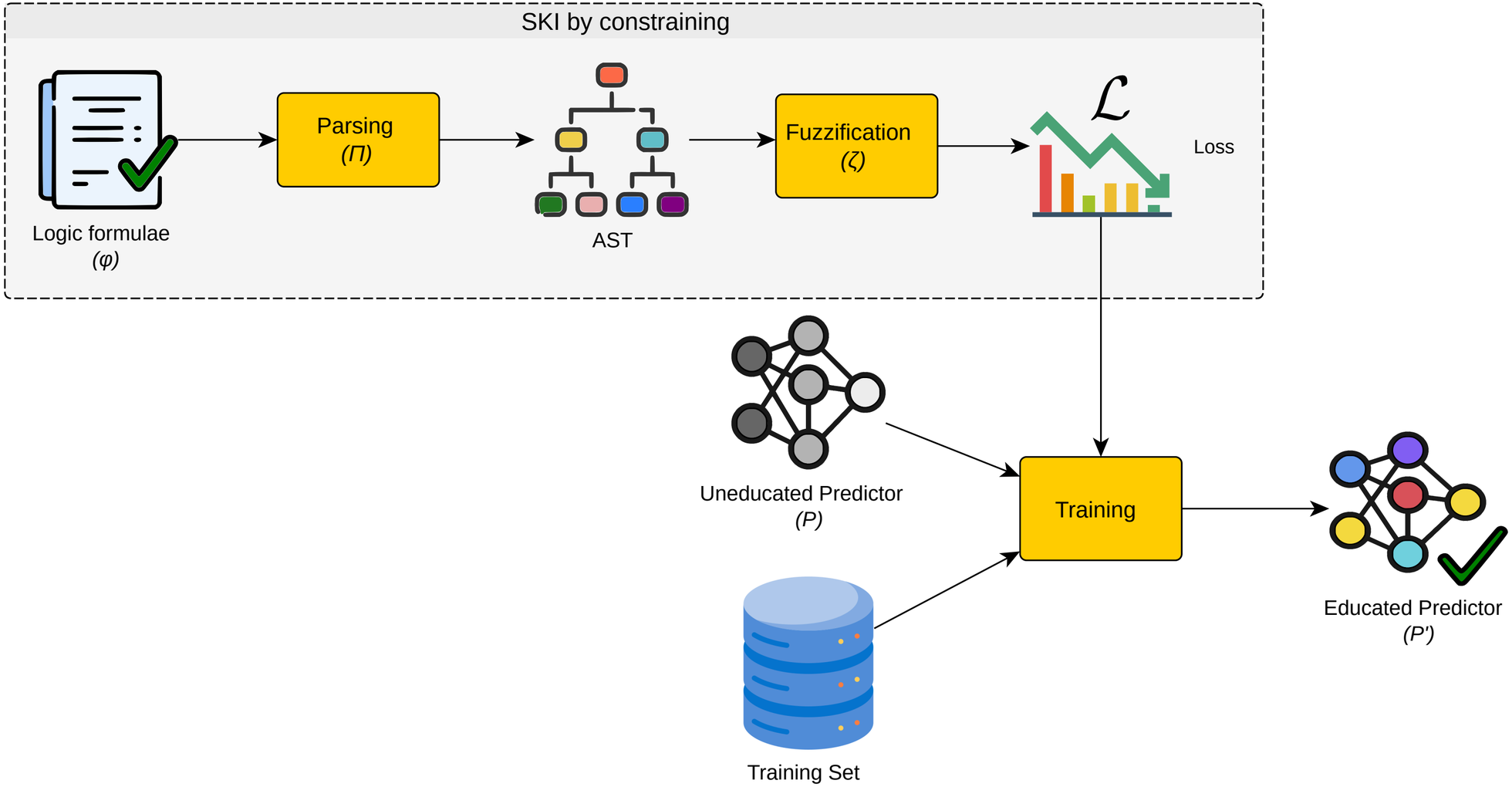
Entities

- **Predictor**: a sub-symbolic model that makes predictions based on input data, usually a neural network
- **Symbolic knowledge**: structured, formal knowledge that can be represented in a symbolic form. The most common forms of symbolic knowledge are
  - *Propositional logic*, simple rules with if-then structure
  - *Datalog*, a subset of first-order logic with no function symbols, only constants and variables
- **Fuzzification**: the process of converting symbolic knowledge into a form that can be used by sub-symbolic predictors, e.g. by assigning degrees of truth to symbolic statements
- **Injector**: the main component that injects symbolic knowledge into the predictor, by modifying its architecture, its training process or by other means

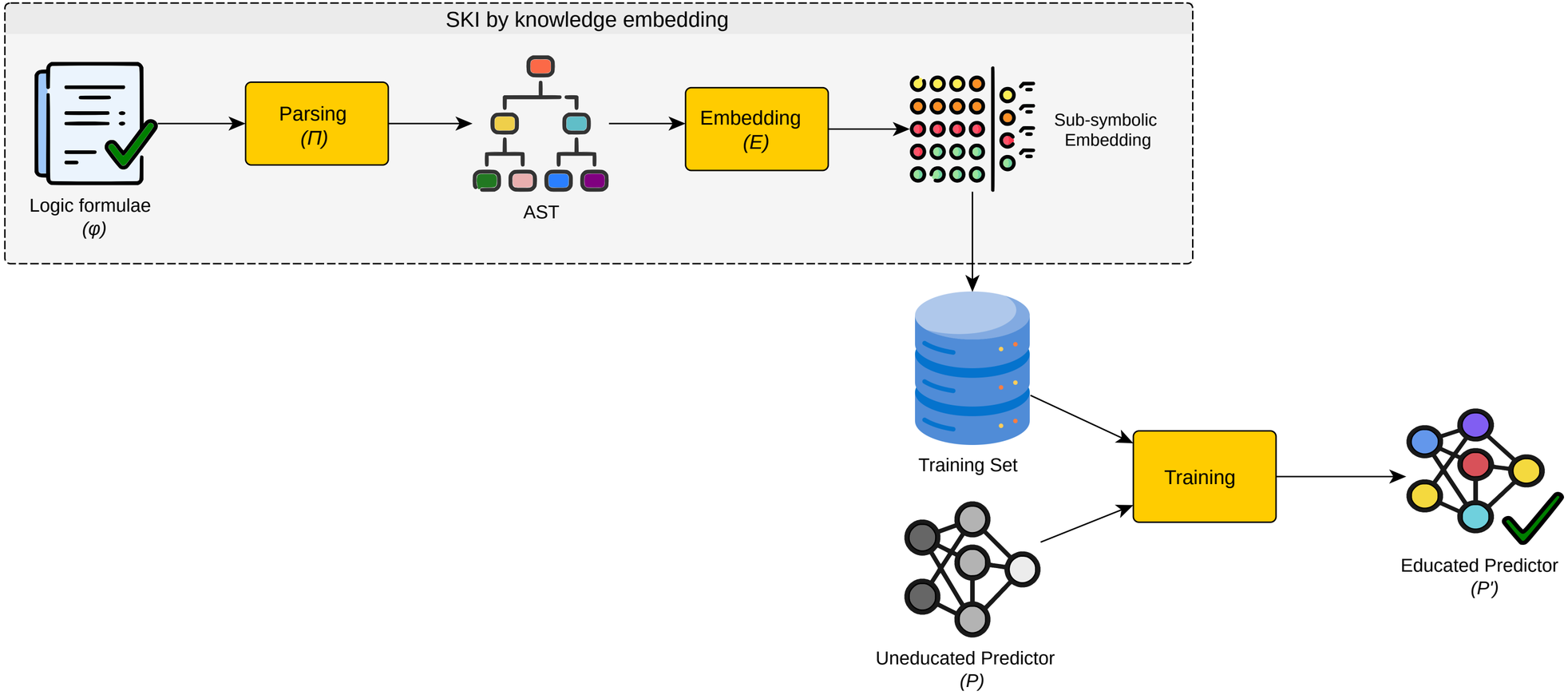
Structuring



Constraining



Embedding


SKI by knowledge embedding
Logic formulae
(φ)
Parsing
(Π)
AST
Embedding
(E)
Sub-symbolic
Embedding
Training Set
Training
Educated Predictor
(P')
Uneducated Predictor
(P)

Knowledge Injection via Network Structuring (KINS)

(ref. Magnini et al., 2023)

Fuzzification

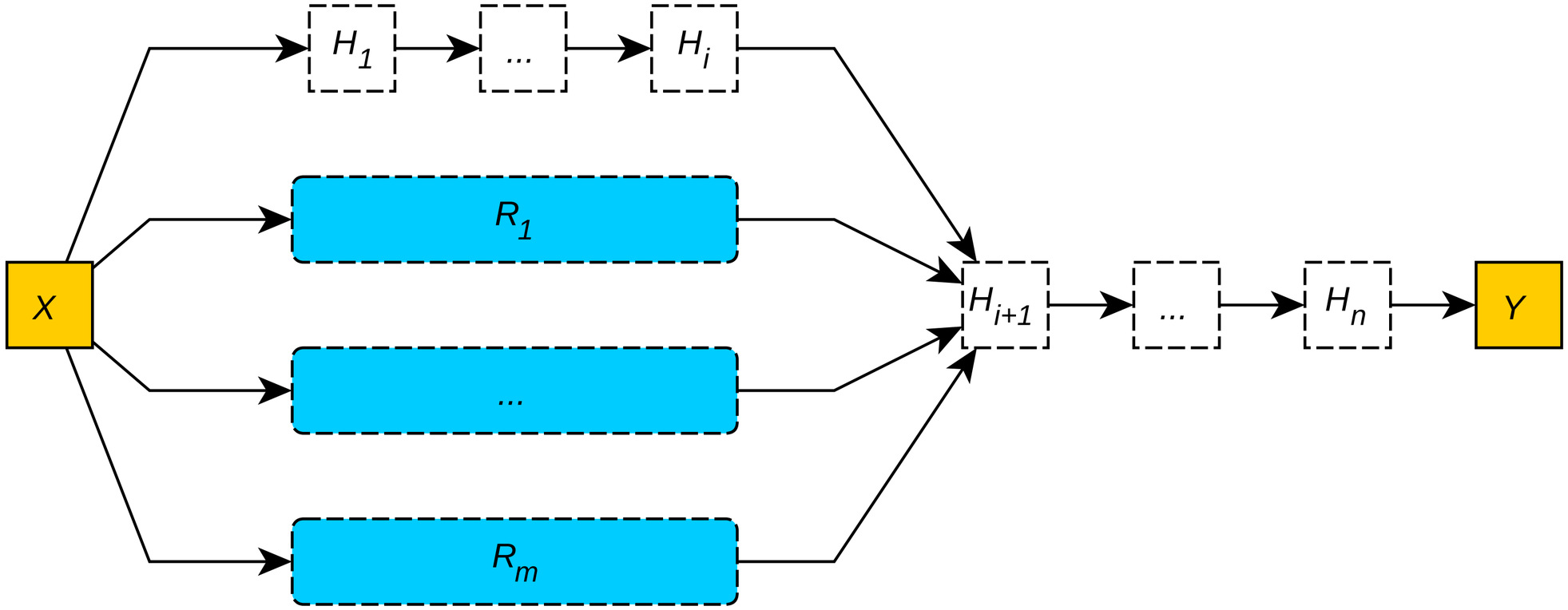
Formula	C. interpretation	Formula	C. interpretation
$[[\neg\phi]]$	$\eta(1-[[\phi]])$	$[[\phi \leq \psi]]$	$\eta(1+[[\psi]]-[[\phi]])$
$[[\phi \wedge \psi]]$	$\eta(\min([[\phi]],[[\psi]]))$	$[[class(\bar{X}, y_i) \leftarrow \psi]]$	$[[\psi]]^*$
$[[\phi \vee \psi]]$	$\eta(\max([[\phi]],[[\psi]]))$	$[[\mathtt{expr}(\bar{X})]]$	$\mathtt{expr}([[\bar{X}]])$
$[[\phi = \psi]]$	$\eta([[\neg(\phi \neq \psi)]])$	$[[\mathtt{true}]]$	1
$[[\phi \neq \psi]]$	$\eta(\lvert[[\phi]]-[[\psi]]\rvert)$	$[[\mathtt{false}]]$	0
$[[\phi > \psi]]$	$\eta(\max(0,\frac{1}{2}+[[\phi]]-[[\psi]]))$	$[[X]]$	x
$[[\phi \geq \psi]]$	$\eta(1+[[\phi]]-[[\psi]])$	$[[k]]$	k
$[[\phi < \psi]]$	$\eta(\max(0,\frac{1}{2}+[[\psi]]-[[\phi]]))$	$[[p(\bar{X})]]^{**}$	$[[\psi_1 \vee \ldots \vee \psi_k]]$

* encodes the value for the i^{th} output ** assuming p is defined by k clauses of the form:

$$p(\bar{X}) \leftarrow \psi_1, \ \ldots, \ p(\bar{X}) \leftarrow \psi_k$$

Injector (pt.1)

Injector (pt. 2)



Knowledge Injection via Lambda Layer (KILL)

(ref. Magnini et al., 2022)

Fuzzification

Formula	C. interpretation	Formula	C. interpretation
$[[\neg\phi]]$	$\eta(1 - [[\phi]])$	$[[\phi \leq \psi]]$	$\eta([[\phi]] - [[\psi]])$
$[[\phi \wedge \psi]]$	$\eta(\max([[\phi]], [[\psi]]))$	$[\mathrm{class}(\bar{X}, y_i) \leftarrow \psi]]$	$[[\psi]]^*$
$[[\phi \vee \psi]]$	$\eta(\min([[\phi]], [[\psi]]))$	$[\mathrm{expr}(\bar{X})]]$	$\mathrm{expr}([[\bar{X}]])$
$[[\phi = \psi]]$	$\eta(\lvert[[\phi]] - [[\psi]]\rvert)$	$[[\texttt{true}]]$	0
$[[\phi \neq \psi]]$	$[[\neg(\phi = \psi)]]$	$[[\texttt{false}]]$	1
$[[\phi > \psi]]$	$\eta(\frac{1}{2} - [[\phi]] + [[\psi]])$	$[[X]]$	x
$[[\phi \geq \psi]]$	$\eta([[\psi]] - [[\phi]])$	$[[k]]$	k
$[[\phi < \psi]]$	$\eta(\frac{1}{2} + [[\phi]] - [[\psi]])$	$[\mathrm{p}(\bar{X})]]^{**}$	$[[\psi_1 \vee \ldots \vee \psi_k]]$

* encodes the penalty for the i^{th} neuron ** assuming predicate p is defined by k clauses of the form:

$$p(\bar{X}) \leftarrow \psi_1, \ \ldots, \ p(\bar{X}) \leftarrow \psi_k$$

Injector (pt.1)

Cost function: whenever the neural network wrongly predicts a class and violates the prior knowledge a cost proportional to the violation is added. In this way the output of the network differs more from the expected one and this affects the back propagation step.

$$Y' = f(Y, \text{cost})$$

$$f = Y \times (\mathbf{1} + \text{cost})$$

$$\text{cost}(X, Y) = \eta(\text{p}(X) - (\mathbf{1} - Y)) \quad (\mathbf{1} - Y \text{ because } 0 \text{ means true})$$

Injector (pt. 2)

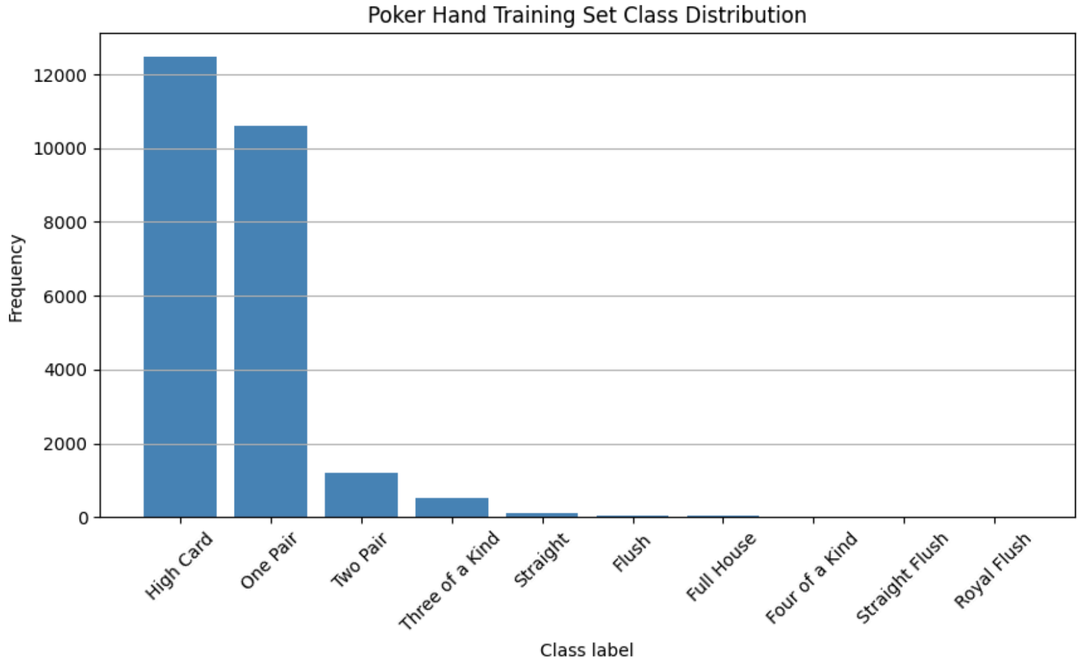
PHDS classification task

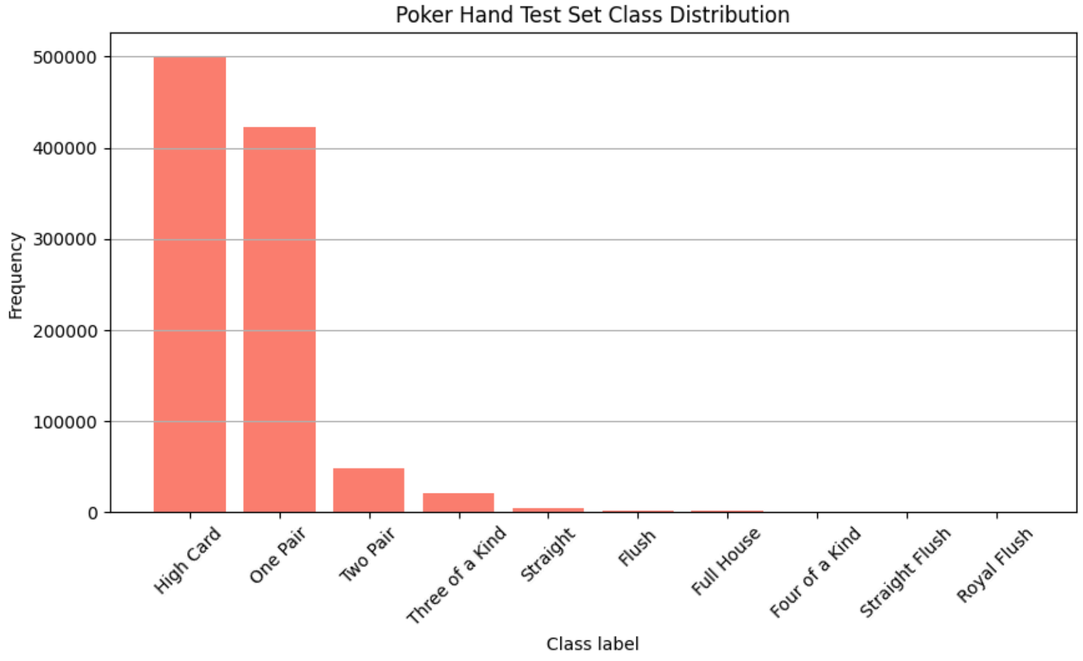
The poker hand data set (PHDS) (cf. Cattral Robert and Oppacher Franz, 2002)

- Each record represents one poker hand
- 5 cards identified by 2 values: suit and rank
- Classes: 10
- Training set: 25,010
- Test set: 1,000,000

id	S1	R1	S2	R2	S3	R3	S4	R4	S5	R5	class
1	1	10	1	11	1	13	1	12	1	1	9
2	2	11	2	13	2	10	2	12	2	1	9
3	3	12	3	11	3	13	3	10	3	1	9
4	4	10	4	11	4	1	4	13	4	12	9
5	4	1	4	13	4	12	4	11	4	10	9
6	1	2	1	4	1	5	1	3	1	6	8
7	1	9	1	12	1	10	1	11	1	13	8
8	2	1	2	2	2	3	2	4	2	5	8
9	3	5	3	6	3	9	3	7	3	8	8
10	4	1	4	4	4	2	4	3	4	5	8
11	1	1	2	1	3	9	1	5	2	3	1
12	2	6	2	1	4	13	2	4	4	9	0
13	1	10	4	6	1	2	1	1	3	8	0
14	2	13	2	1	4	4	1	5	2	11	0
15	3	8	4	12	3	9	4	2	3	2	1

An unbalanced dataset





Logic rules to inject (pt. 1)

Class	Logic Formulation
Pair	$class(R_1, ..., S_5, pair) \leftarrow pair(R_1, ..., S_5)$
	$pair(R_1, ..., S_5) \leftarrow R_1 = R_2$
	$pair(R_1, ..., S_5) \leftarrow R_1 = R_3$
	$pair(R_1, ..., S_5) \leftarrow R_1 = R_4$
	$pair(R_1, ..., S_5) \leftarrow R_1 = R_5$
	$pair(R_1, ..., S_5) \leftarrow R_2 = R_3$
	$pair(R_1, ..., S_5) \leftarrow R_2 = R_4$
	$pair(R_1, ..., S_5) \leftarrow R_2 = R_5$
	$pair(R_1, ..., S_5) \leftarrow R_3 = R_4$
	$pair(R_1, ..., S_5) \leftarrow R_3 = R_5$
	$pair(R_1, ..., S_5) \leftarrow R_4 = R_5$

Logic rules to inject (pt. 2)

Class	Logic Formulation
Two Pairs	$class(R_1, ..., S_5, two) \leftarrow two(R_1, ..., S_5)$
	$two(R_1, ..., S_5) \leftarrow R_1 = R_2 \wedge R_3 = R_4$
	$two(R_1, ..., S_5) \leftarrow R_1 = R_3 \wedge R_2 = R_4$
	$two(R_1, ..., S_5) \leftarrow R_1 = R_4 \wedge R_2 = R_3$
	$two(R_1, ..., S_5) \leftarrow R_1 = R_2 \wedge R_3 = R_5$
	$two(R_1, ..., S_5) \leftarrow R_1 = R_3 \wedge R_3 = R_5$
	$two(R_1, ..., S_5) \leftarrow R_1 = R_5 \wedge R_2 = R_3$
	$two(R_1, ..., S_5) \leftarrow R_1 = R_2 \wedge R_4 = R_5$
	$two(R_1, ..., S_5) \leftarrow R_1 = R_4 \wedge R_2 = R_5$
	$two(R_1, ..., S_5) \leftarrow R_1 = R_5 \wedge R_2 = R_4$
	$two(R_1, ..., S_5) \leftarrow R_1 = R_3 \wedge R_4 = R_5$
	$two(R_1, ..., S_5) \leftarrow R_1 = R_4 \wedge R_3 = R_5$
	$two(R_1, ..., S_5) \leftarrow R_1 = R_5 \wedge R_3 = R_4$
	$two(R_1, ..., S_5) \leftarrow R_2 = R_3 \wedge R_4 = R_5$
	$two(R_1, ..., S_5) \leftarrow R_2 = R_4 \wedge R_3 = R_5$
	$two(R_1, ..., S_5) \leftarrow R_2 = R_5 \wedge R_3 = R_4$

Logic rules to inject (pt. 3)

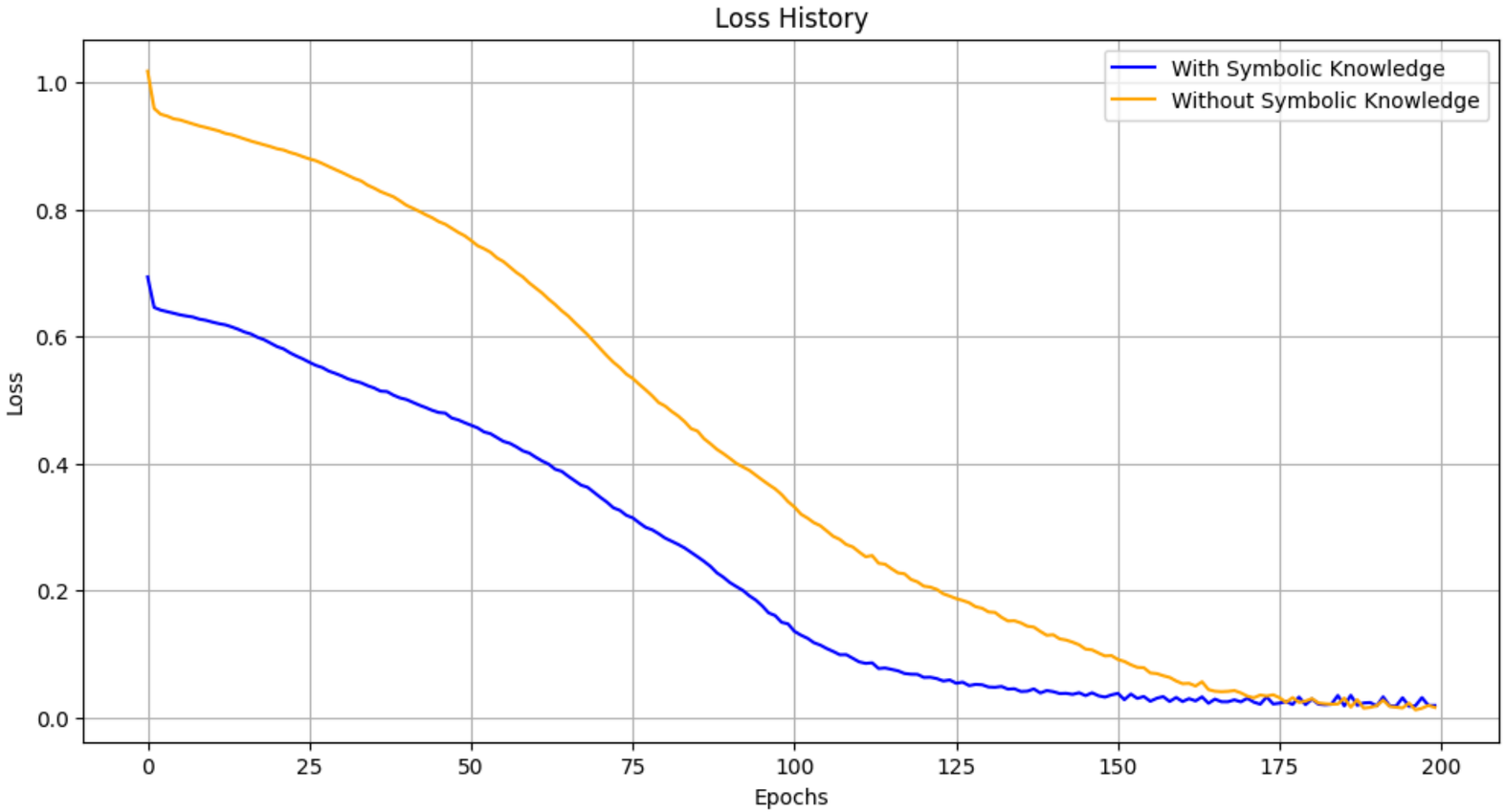
Class	Logic Formulation
Three of a Kind	$class(R_1, ..., S_5, three) \leftarrow three(R_1, ..., S_5)$
	$three(R_1, ..., S_5) \leftarrow R_1 = R_2 \wedge R_1 = R_3$
	$three(R_1, ..., S_5) \leftarrow R_1 = R_2 \wedge R_1 = R_4$
	$three(R_1, ..., S_5) \leftarrow R_1 = R_2 \wedge R_1 = R_5$
	$three(R_1, ..., S_5) \leftarrow R_1 = R_3 \wedge R_1 = R_4$
	$three(R_1, ..., S_5) \leftarrow R_1 = R_3 \wedge R_1 = R_5$
	$three(R_1, ..., S_5) \leftarrow R_1 = R_4 \wedge R_1 = R_5$
	$three(R_1, ..., S_5) \leftarrow R_2 = R_3 \wedge R_2 = R_4$
	$three(R_1, ..., S_5) \leftarrow R_2 = R_3 \wedge R_2 = R_5$
	$three(R_1, ..., S_5) \leftarrow R_2 = R_4 \wedge R_2 = R_5$
	$three(R_1, ..., S_5) \leftarrow R_3 = R_4 \wedge R_3 = R_5$
Flush	$class(R_1, ..., S_5, flush) \leftarrow flush(R_1, ..., S_5)$
	$flush(R_1, ..., S_5) \leftarrow S_1 = S_2 \wedge S_1 = S_3 \wedge S_1 = S_4 \wedge S_1 = S_5$

Training the models (pt. 1)

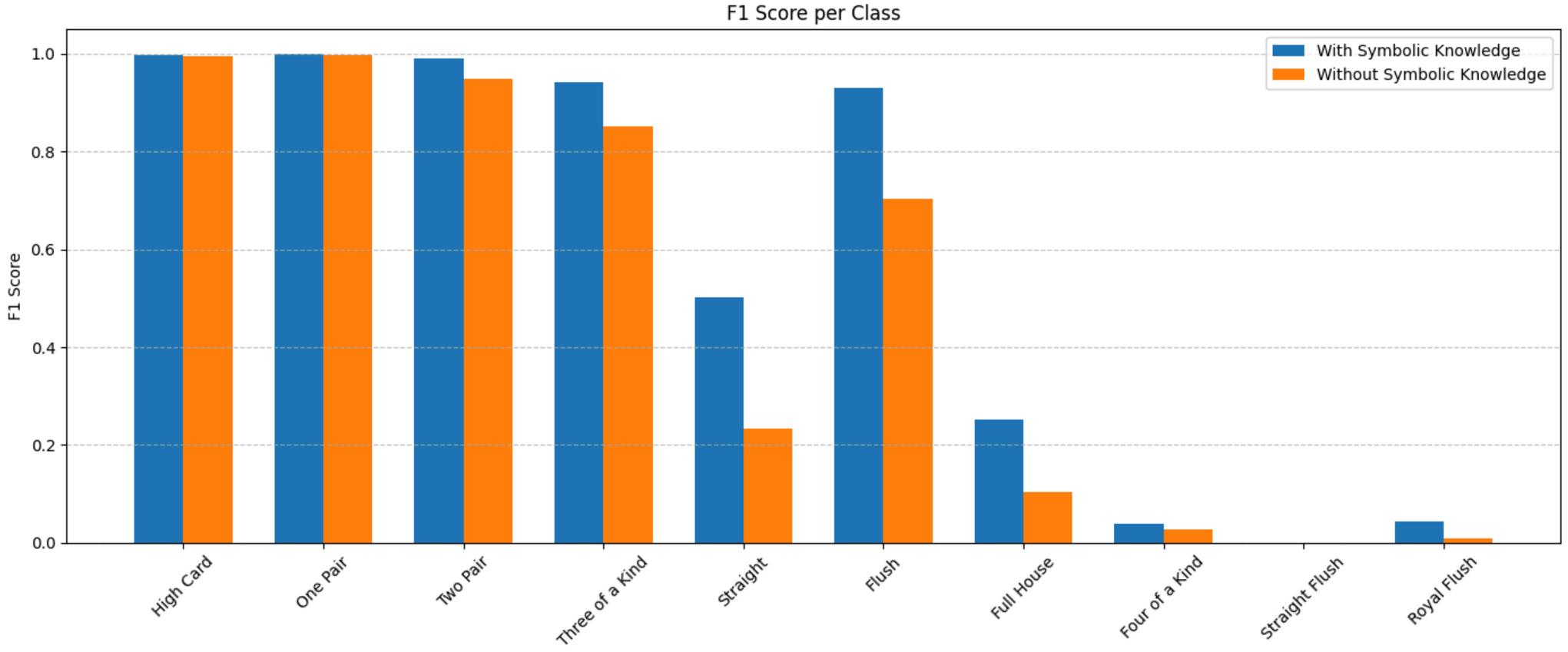
```python
class PokerNet(nn.Module):
    def __init__(self):
        super().__init__()
        self.model = nn.Sequential(
            nn.Linear(FEATURE_NUMBER, HIDDEN_SIZE),
            nn.ReLU(),
            nn.Linear(HIDDEN_SIZE, HIDDEN_SIZE),
            nn.ReLU(),
            nn.Linear(HIDDEN_SIZE, CLASS_NUMBER)
        )

    def forward(self, x):
        return self.model(x)
```

```python
def rule_high_card(x_batch_orig, pred_logits):
    ranks = x_batch_orig[:, 1::2].int()  # Extract ranks from the input
    num_pairs = count_occurrences(ranks, 2)  # Count occurrences of pairs
    is_high_card = (num_pairs == 0)  # Check if there are no pairs
    prob_high_card = torch.softmax(pred_logits, dim=1)[:, 0]  # Probability of "H
    penalty = ((1 - prob_high_card) ** 2) * is_high_card.float()  # Calculate pena
    return penalty.mean()

def rule_one_pair(x_batch_orig, pred_logits):
    ranks = x_batch_orig[:, 1::2].int()  # Extract ranks from the input
    num_pairs = count_occurrences(ranks, 2)  # Count occurrences of pairs
    is_one_pair = (num_pairs == 1)  # Check if there is exactly one pair
    prob_one_pair = torch.softmax(pred_logits, dim=1)[:, 1]  # Probability of "On
    penalty = ((1 - prob_one_pair) ** 2) * is_one_pair.float()  # Calculate penal
    return penalty.mean()
```

Training the models (pt. 2)



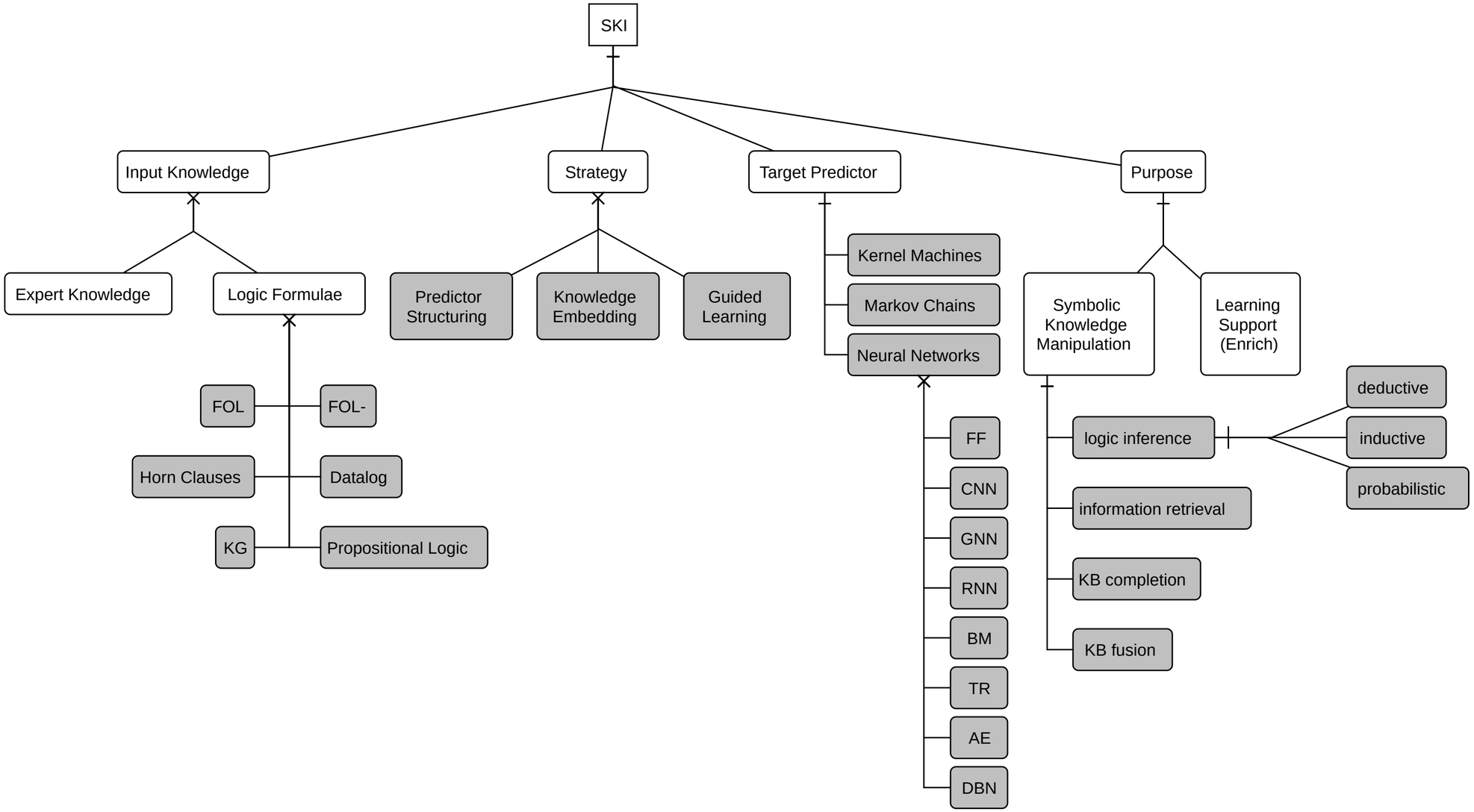
Results


F1 Score per Class
With Symbolic Knowledge
Without Symbolic Knowledge
F1 Score
1.0
0.8
0.6
0.4
0.2
0.0
High Card
One Pair
Two Pair
Three of a Kind
Straight
Flush
Full House
Four of a Kind
Straight Flush
Royal Flush

Jupyter notebook available here

github.com/MatteoMagnini/demo-2025-woa-nesy/blob/master/notebook/injection.ipynb

Taxonomy of SKI methods (pt. 1)



Taxonomy of SKI methods (pt. 2)

- **input knowledge**: how is the knowledge to-be-injected represented?
  - commonly, some sub-set of first-order logic (FOL)
- **target predictor**: which predictors can knowledge be injected into?
  - mostly, neural networks
- **strategy**: how does injection actually work?
  - *guided learning*: the input knowledge is used to *guide the training* process
  - *structuring*: the *internal* composition of the predictor is *(re-)structured* to reflect the input knowledge
  - *embedding*: the input knowledge is *converted* into numeric array form
- **purpose**: why is knowledge injected in the first place?
  - *knowledge manipulation*: improve / extend / reason about symbol knowledge—subsymbolically
  - *learning support*: improve the sub-symbolic predictor (e.g. speed, size, etc.)

Discussion

Notable remarks

- Knowledge should express relations about input-output pairs
- embedding implies *extensional* representation of knowledge
- guided learning and structuring support *intensional* knowledge
- propositional knowledge implies binarising the I/O space

Limitations

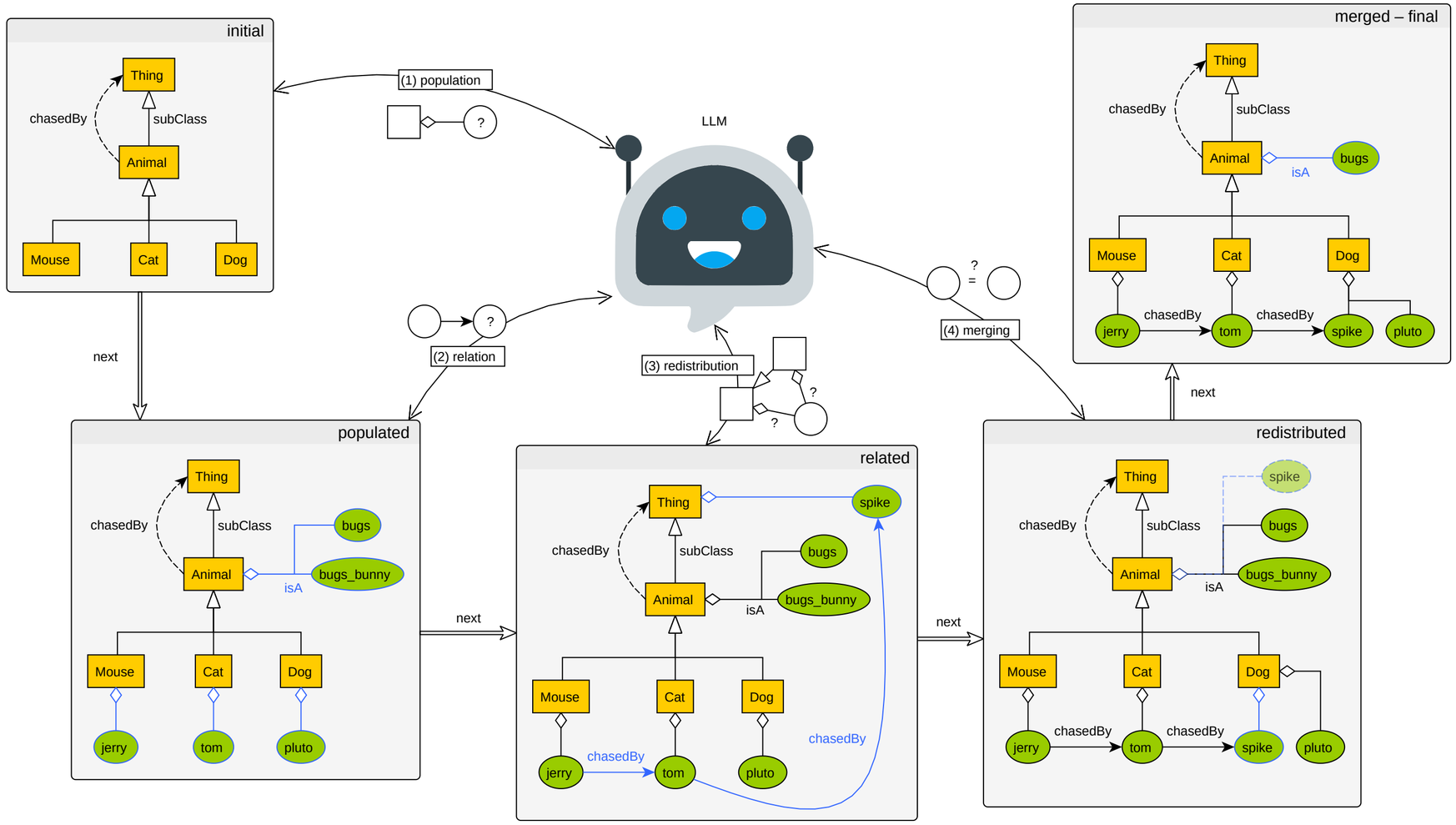
- Recursive data structures are natively not supported
- extensional representation cost storage
- guided learning works poorly with lacking data

NeSy applications with LLMs

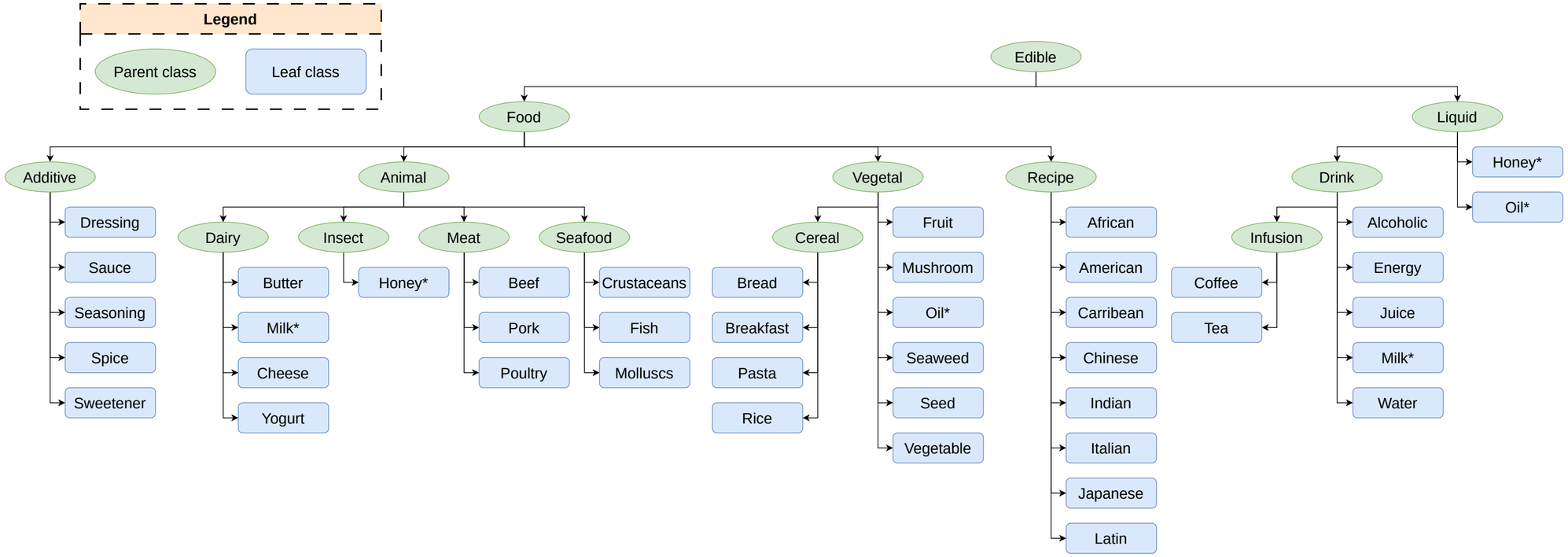
Last recent works on Neural-Symbolic AI involve Large Language Models

LLMs as oracles for instantiating ontologies with domain-specific knowledge

(ref. Ciatto et al., 2025)



Experiments



Results

Table 4
Feature-based comparison between KGFiller and state-of-the-art KG construction methods. The arrow (→) denotes the best-featured method from the literature, namely Harvest, which we compare with KGFiller under a performance-based perspective.

Method	Document Free	Training Free	Construction	Prompt Templating	Consistency
[70]	✗	✗	✓	✗	✗
[58]	✗	✗	✓	✗	✗
[55]	✗	✗	✓	✗	✗
[46]	✗	✗	✗	✗	✗
[47]	✗	✗	✗	✗	✗
[54]	✗	✗	✓	✗	✗
[56]	✗	✗	✓	✗	✗
[49]	✗	✗	✗	✗	✗
[50]	✗	✗	✗	✗	✗
[51]	✗	✗	✗	✗	✗
[53]	✗	✓	✓	✗	✗
[48]	✗	✗	✗	✗	✗
[57]	✗	✓	✓	✓	✗
→[59]	✓	✓	✓	✓	✗
[71]	✓	✗	✓	✓	✗
[72]	✓	✗	✓	✓	✗
Ours (KGFiller)	✓	✓	✓	✓	✓

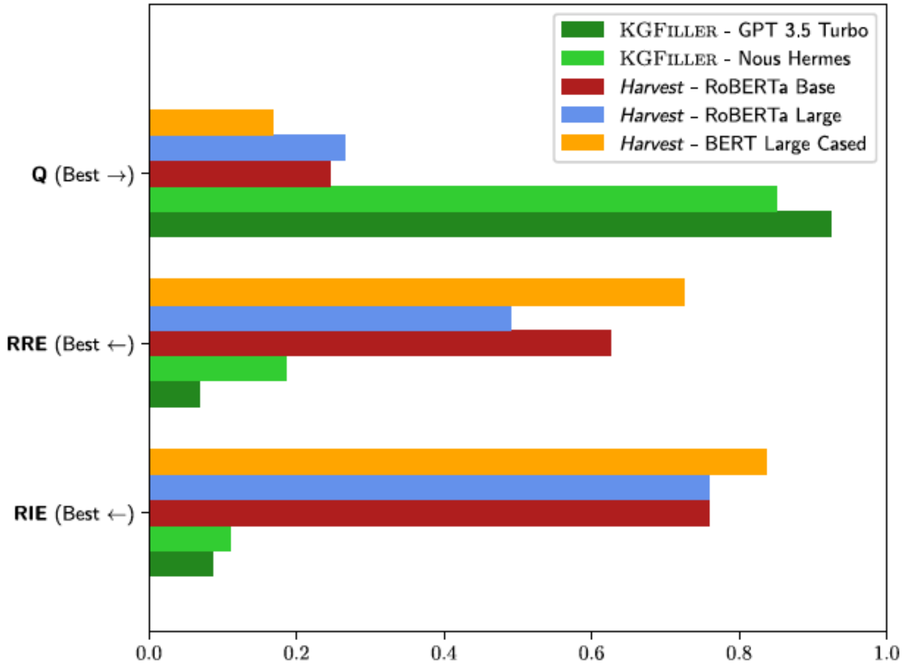
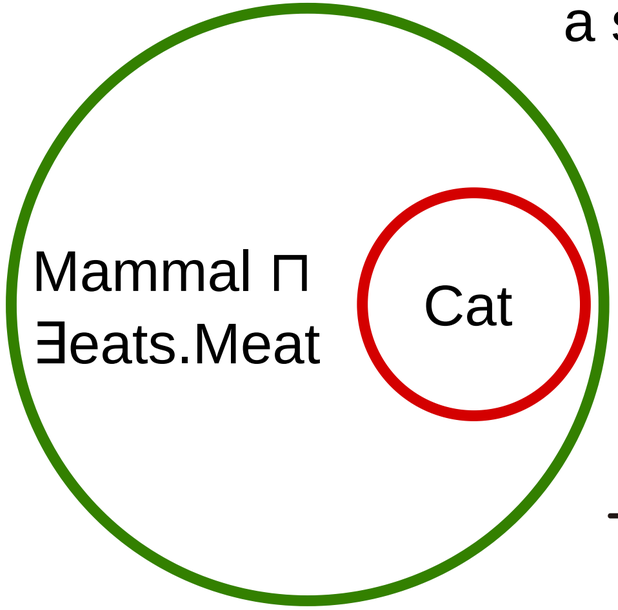


Fig. 4. Comparison of the performance of Harvest [59] and KGFiller w.r.t. the task of populating our food ontology (cf. Section 4.1). For KGFiller, the best-performing closed-source and open-source LLM models are considered — respectively GPT 3.5 Turbo and Nous Hermes.

Actively Learning EL Terminologies from LLMs (pt. 1)

(ref. Magnini et al., 2025)

Can **cat** be considered a subcategory of mammal that eats meat?

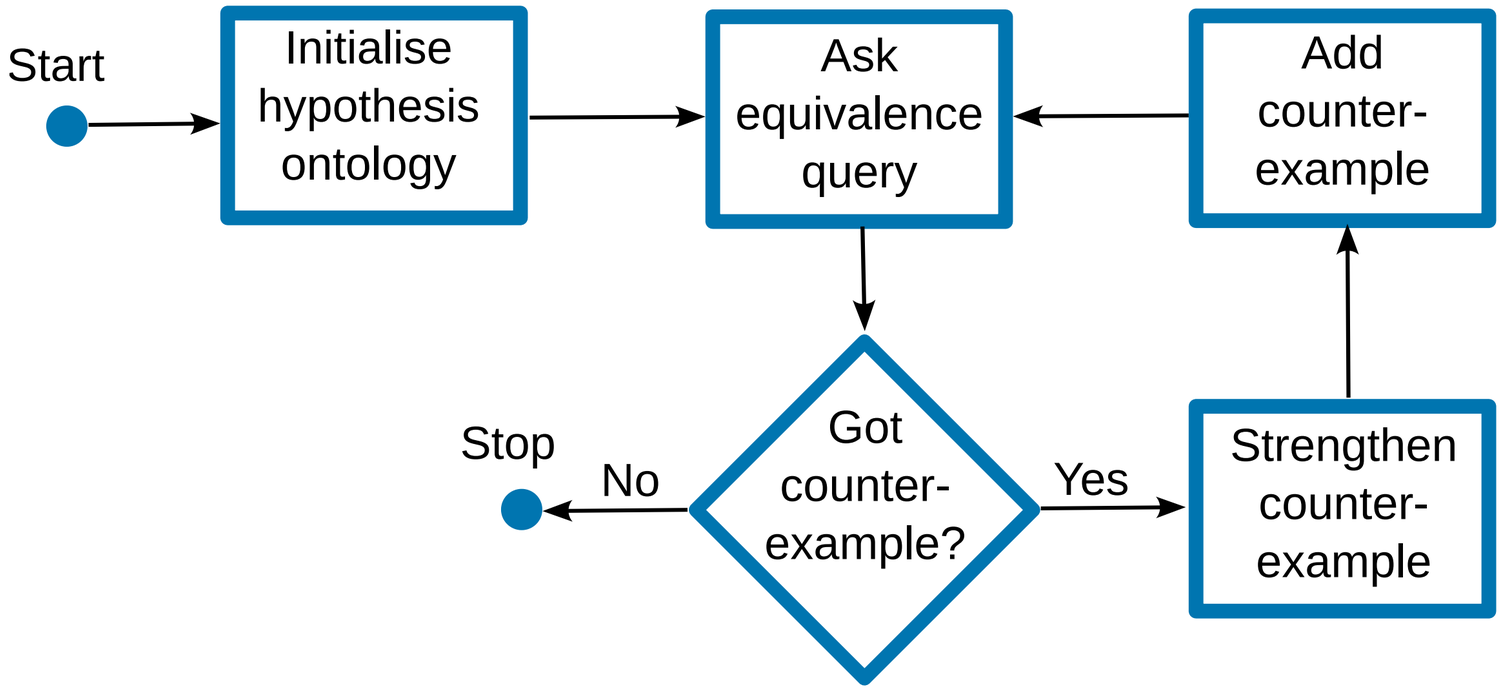


An equivalence query is **true** or **false**. If false, the teacher provides a **counterexample**.

Thing
- Game
  - Ball Game
    - Football
- Person
  - Manager
  - Player
    - Football Player
- Team
  - Football Team

Football Player ⊑ ∃plays.Game

Actively Learning EL Terminologies from LLMs (pt. 2)



Experiments and Results (pt. 1)

Ontology	N_C	N_R	Log. Ax.	PAC Sample	Poss. Ax.
Animals	17	4	12	542	6,936
Cell	22	0	24	1,119	10,164
Football	10	3	9	341	1,500
Generations	20	4	18	847	10,800
University	7	3	4	139	588

Ontology statistics and PAC sample sizes with $\epsilon = 0.2$ and $\gamma = 0.1$. N_C and N_R are the number of concept and role names occurring in the ontologies.

Ontology	Accuracy	Recall	Precision	F1-Score
Animals	0.737	0.858	0.381	0.428
Cell	0.391	0.733	0.206	0.284
Football	0.553	0.890	0.422	0.477
Generations	0.691	0.658	0.564	0.476
University	0.622	0.629	0.313	0.302

Results of ExactLearner+LLM grouped by ontologies.

Experiments and Results (pt. 2)

Model	Accuracy	Recall	Precision	F1-Score
Llama2 (13b)	0.521	0.71	0.294	0.314
Llama3 (8b)	0.43	0.947	0.218	0.333
Mistral (7b)	0.741	0.747	0.45	0.49
Mixtral (47b)	0.705	0.611	0.547	0.436

Results of ExactLearner+LLM grouped by models.

Prompt Type	Accuracy	Recall	Precision	F1-Score
M. OWL Syntax	0.34	0.93	0.165	0.262
Natural Language	0.751	0.811	0.414	0.511
A. M. OWL Syntax	0.537	0.767	0.326	0.347
A. Natural Language	0.767	0.506	0.603	0.454

Results of ExactLearner+LLM grouped by prompts.

I hope you enjoyed the talk!

Let’s keep in touch!

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